On Outsourced Abatement Services: Market Power and Efficient Environmental Regulation

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Abstract

In this paper, we consider competitive polluting firms that outsource their abatement activity to an upstream imperfect competitive eco-industry. In this case, we show that a standard environmental policy based on a Pigouvian tax or a pollution permit market reaches the first-best outcome. A polluting firm buying these services at a given price does not really care how pollution is reduced. This induces purchasing behavior which is, at least partially, strongly elastic and therefore strongly reduces upstream market power. This argument is first illustrated with an upstream monopoly selling eco-services to a representative polluting firm. We then progressively extend the result to heterogeneous downstream polluters and heterogeneous upstream Cournot competitors.

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1. Introduction

The Environmental Goods and Services Sector (EGSS hereafter) is currently of major interest to policy makers and is given wide coverage by most statistical institutes.[1] While acknowledging that the EGSS remains moderate in size (around 2% on GDP in both Europe and the US), these institutes stress the sector's exceptional growth rate, its capacity to generate new job opportunities and its export performance. For instance, estimates for the European Union between 2000 and 2011 show an increase in EGSS output per unit of GDP of 50 % , whereas employment grew at around 40%. The strong development of the EGSS over the past two decades is basically due to the implementation, at least in developed countries, of more stringent environmental policies. On these new markets, traditional polluting industries purchase technologies, goods or services which reduce their emissions while simultaneously avoiding, at least partially, the additional costs these environmental policies entail. This outsourcing of, say, pollution abatement typically has no consequences on the implementation of these policies, insofar as the markets remain competitive. However, it is widely acknowledged that EGSS is controlled by global firms like CH2M Hill, Veolia Environmental Services, Vivendi Environment or Suez Environnement. The Ecorys report [10] on the European EGSS even states that 10% of the companies account for almost 80% of the operating revenue.

In this case, with -perfect competition in the downstream polluting industry and market power in the upstream eco-industry-, fewer abatement solutions are traded than if there were perfect eco-industry competition. Thus, the regulator has an incentive to set the Pigouvian tax above the marginal damage, to compensate. The strength of this distortion is closely related to the degree of market power of the eco-industry, hence - as is usual in industrial organization - to the elasticity of demand. This demand is endogenous: it is deduced from the cost minimization strategy of each polluting firm facing the environmental regulation. This means that the demand for abatement goods is mainly explained by the performance of the polluting firm with respect to this specific input, i.e. its marginal productivity to abate pollution. The elasticity of this demand is thus related to the change in marginal pollution reduction.

This is especially true of end-of-pipe remediation activities. In this case, abatement does not modify the production process and therefore does not affect the marginal contribution to pollution of each final product. This remediation simply reduces emissions and therefore the costs entailed by a Pigouvian tax (or the purchase of tradeable pollution permits). As a result, the literature claims that each polluting firm purchases abatement goods until the tax avoidance of the last unit of abatement good is equal to its price.

[1]Even though some methodological problems remain (see the UNEP report, [36]), several empirical studies have recently sought to quantify the EGSS. For instance, the Canadian statistical institute [35] conducts a biennial survey of the EGSS (http: //www.statcan.gc.ca /eng /survey /business /1209). In Europe, Eurostats has initiated a study over 28 member states (see http: //ec.europa.eu /eurostat /statistics-explained /index.php /Environmental_goods_and_services_sector) based on a methodology described in Eutostats [12]. In 2010, the US department of commerce published a survey called "Measuring the Green Economy" [37].

However, since the tax rate is constant, this suggests that each dirty firm perfectly knows (or even controls) the pollution abatement technology and incorporates this knowledge into his tax avoidance strategy. This is typically the case for a large number of goods such as filters, scrubbers or incinerators that help to abate pollution. Once they have used these products, dirty firms quickly recognize that they can reduce emissions. But the polluting by-products of the economic activity may simply be sold to another firm, which operates the decontamination. This occurs, for instance, with waste treatment, especially hazardous waste, with water sewerage and treatment, or with most remediation and clean-up operations. Provided the operation is performed by an eco-service firm at a given market price per unit of pollutants, the dirty firm is not really concerned by how it is performed. The dirty firm simply outsources buying a decontamination service from another firm.

This last observation on what can be called marketable environmental services[2] fundamentally modifies the polluter's purchasing behavior: when he buys abatement services, he only makes a trade-off between the price of this service and the cost of non-compliance with an environmental regulation, i.e., without regard for the efficiency of the decontamination equipment. For this polluting firm, the marginal productivity of the abatement service is equal to one. This means that the elasticity of the demand becomes infinite, at least over a given range of this function. This at least partially offsets upstream market power, raising the question of whether the first-best outcome can be reached even under imperfect competition in the eco-industry.

The main purpose of this paper is to provide a positive answer to this question. We assume marketable eco-services which contribute to end-of-pipe pollution abatement. In this case, we show that the regulator has the opportunity to implement the first-best outcome even if the upstream eco-service market is imperfectly competitive.

This result is mainly driven by the reversal of two assumptions that are made by almost all the papers dealing with upstream imperfect competition and end-of-pipe abatement. Their authors usually assume that the dirty firm reduces pollution by an amount $A(a)$, a concave function of the quantity of abatement good a, these goods being produced at a constant marginal cost by the eco-industry. Here, we assume that the polluting firm outsources the decontamination activity, in other words, decides on the amount A of abatement at the current price p_A of this service (i.e. bears a constant unit cost p_A), while the clean-up activity is performed by an upstream firm characterized by convex costs (or, equivalently, a concave abatement function). This assumption reversal fundamentally modifies the polluter's purchasing behavior. For instance if a Pigouvian tax is imposed, he either (i) chooses to pay the tax if the price of the abatement service is higher, (ii) decides, in the opposite case, to fully abate the pollution produced at the firm's equilibrium

[2] This term should not be misinterpreted. While environmental services represent more than 40% of the activity of the EGSS (Sinclair-Desgagné [34] table 2), our narrow definition only covers clean-up activities which are outsourced. If the object of the transaction forms part of the production process of the dirty firm, it is not, in our terminology, considered a service.

production level, or (iii) is indifferent between the two if the price and the tax are equal. This implies that the demand for eco-services becomes perfectly elastic over a range of quantities which depends on the tax level, so that any monopoly selling these services loses - at least partially - his market power. If the regulator is able to set a tax level such that the monopoly solution belongs to this range of quantities, he clearly destroys upstream monopoly power. He therefore has the opportunity, if the downstream polluting market is competitive, to reach the first-best allocation. Owing to the structure of demand for eco-services, this situation occurs when the monopoly has an incentive to set the highest price at which demand is positive, i.e. the tax level, and to supply, due to marginal cost concerns, a quantity of services lower than required for full pollution abatement. In this situation, the abatement service price equates tax level to marginal cost. If the efficient abatement level does not require full abatement, it remains for the regulator to set the Pigouvian tax equal to optimal marginal damage in order to obtain the first-best outcome.

This paper proceeds as follows. Section 2 gives a presentation of the background literature as related to our question. Section 3 describes the model. Section 4 develops the argument in the simplest setting: homogeneous competitive polluters, an eco-service monopoly and an optimal abatement level that does not require full abatement. In Section 5 we relax several assumptions of this basic case. We show that our result can easily be extended to (i) include the "boundary" solutions corresponding to full efficient pollution abatement, (ii) take into account regulation by a pollution permit market, (iii) consider polluters who are heterogeneous with regard to their production costs and to their emissions, and (iv) introduce Cournot competition in an eco-industry composed of firms with heterogenous production costs for abatement goods. Concluding remarks are made in Section 6 and technical proofs are relegated to an appendix.

2. Background literature

The literature on the EGSS essentially focuses on two topics. Either it examines the incentives provided by environmental policy instruments for the development and the diffusion of new abatement technology (see Requate [28] for an overview). Or it explores the consequences of imperfect competition in a mature eco-industry selling abatement goods to a polluting sector (see Sinclair-Desgagné [34] for a general discussion).

In the first type of literature, not all contributions explicitly consider an EGSS, since this requires innovation to be a private good. Studies often consider an innovative firm investing in R&D to obtain a patent on a pollution-reducing new technology. Within this framework, taxes and tradeable permits are compared in terms of performance under various settings. Denicolo [9] and Requate [29] make these comparisons under different timing and commitment regimes. A threat of imitation is introduced by Fischer et al. [13], while Perino [25] studies green horizontal innovation, where new technologies reduce pollution of one type while causing a new type of damage. More recently, Perino and Requate [27] study the relationship between policy stringency and the rate of adoption of an abatement technology. Moreover, most of these papers never introduce imperfect competition in the upstream eco-industry, except Perino [26]. He addresses the question of technology adoption when a competitive and polluting industry purchases this abatement technology from a monopolistic upstream industry.

In contrast, the second type of literature, which is closer to our contribution, takes as given the existence of imperfect competition in the eco-industry and focuses on the provision of abatement goods rather than the adoption of a new technology. The second-best regulation policy is explored under varying instruments. Greaker [15] and Greaker and Rosendahl [16] introduce emission standards. Schwartz and Stahn [33] explore the case of tradable pollution permits, while Endres and Friehe [11] examine the impact of environmental liability laws. However, most of the papers introduce a Pigouvian tax, in line with David and Sinclair-Desgagné [6] and Canton et al. [5], the former introducing imperfect competition upstream while the latter studies imperfect competition both upstream and downstream. Both point out that under the assumption of constant marginal damage, if there is perfect competition downstream and upstream Cournot competition, then the second-best Pigouvian tax must be higher than this constant marginal damage. They explain this distortion by the upstream degree of market power. Other papers add industrial organization arguments, like upstream entry (David et al. [8]), merger in the eco-industry (Canton et al. [4]) or R&D cooperation (Nimubona and Benchekroun [23]).

Our contribution is also close to Nimubona and Sinclair-Desgagné [24], who initiate a discussion about internal abatement effort and external procurement of abatement facilities. Although they depart from the Katsoulacos and Xepapadeas [19] view of end-of-pipe pollution, according to which emission abatement does not modify the production process, they assume, as in most papers, that this activity has decreasing returns for the polluting firm. This excludes the case in which the remediation activity is totally outsourced to the eco-industry.[3] In this case, contrary to David and Sinclair-Desgagné [7], who introduce both Pigouvian tax and subsidy, we will see later that the first-best outcome can be obtained with only one instrument.

3. A basic model of environmental services

This section describes the relationship between a polluting industry and an eco-service industry. The main assumptions are spelt out and the first-best allocation is identified.

3.1. The main assumptions

We consider a **polluting industry** first characterized by a representative price-taking firm and later by firms heterogeneous in both production costs and emissions. The current price of the consumption good is P_Q, and the production cost associated with an output level, Q, is denoted as $c(Q)$. This cost function is assumed twice differentiable, strictly increasing and convex, $c' > 0$, $c'' > 0$, inaction is allowed, $c(0) = 0$, and the usual boundary

[3] In fact, our end-of-pipe emission reduction technology can be viewed as a particular case of the Katsoulacos and Xepapadeas [19] emission function in which the abatement good has constant returns to scale. To the best of our knowledge, this case has not been explored, probably for technical reasons: standard differential calculus does not really apply and corner solutions emerge.

conditions are introduced, $c'(0) = 0$ and $\lim_{Q\to+\infty} c'(Q) = +\infty$. We assume that this activity is polluting, but apply two additional restrictions. First, as is common in the eco-industry literature, we assume that the pollution is end-of-pipe, meaning that abatements do not modify the production process and therefore do not affect the quantity of pollution imputable to each unit produced. We denote by $\varepsilon(Q)$ this quantity of pollution. This function is twice differentiable, strictly increasing and convex, $\varepsilon' > 0$, $\varepsilon'' > 0$, and satisfies $\varepsilon(0) = 0$, $\varepsilon'(0) = 0$ and $\lim_{q\to+\infty} \varepsilon'(q) = +\infty$. Secondly, we also assume that these noxious by-products, $\varepsilon(Q)$, can be transferred to an external firm in charge of sanitation. This kind of eco-service includes waste disposal, water treatment, or remediation activities like soil sanitation. Of course, it is always possible that the production process may generate other forms of pollution, like air pollution which are managed by the dirty firm itself, for instance by using specific scrubbers produced by the eco-industry. But for these abatement goods, we are back to the standard literature. For simplicity, we therefore assume that the polluting firm only has the opportunity to sell a proportion A of its noxious by-products to a specialized external firm at a given market price p_A. The remaining pollution is therefore $E(Q, A) = \max\{\varepsilon(Q) - A, 0\}$.

A non-competitive **eco-industry** carries out the clean-up. This industry is initially characterized by a monopoly, and later by heterogeneous firms in Cournot competition. We implicitly assume that this activity has decreasing returns and requires inputs to be obtained on competitive markets. We can therefore summarize this activity by a cost function $\kappa(A)$ which is twice differentiable, strictly increasing and convex, $\kappa' > 0$, $\kappa'' > 0$. We also assume that inaction is possible, $\kappa(0) = 0$; however, to ensure that the eco-industry is active, we assume that $\kappa'(0) = 0$.[4]

The **environmental damage** induced by the remaining emissions E of the polluting sector is measured by a standard damage function $D(E)$. As usual, this function is assumed to be increasing and convex, $D' > 0$ and $D'' \geq 0$. We also say that without emissions there is no damage, $D(0) = 0$, and assume, for convenience, that there is no marginal damage at a zero-emission level, $D'(0) = 0$. This last assumption combined with the definition of the remaining emissions ensures that full abatement never occurs at an efficient allocation.[5] However, it excludes the case of constant marginal damage, which is why we show in Section 4.1 that this restriction is not central to our main argument.

Finally, to close the model, we introduce an **inverse demand** for the polluting goods $P(Q)$. This function is decreasing, $P' < 0$, and verifies that $\lim_{Q\to0} P(Q) = +\infty$ and $\lim_{Q\to+\infty} P(Q) = 0$.

[4]A discussion about the emergence of an eco-industry related to the fact that $\kappa'(0) > 0$ can be found in Canton et al. [5].

[5]If the marginal damage at zero is high enough and/or the marginal abatement cost is not too excessive, the end-of-pipe pollution assumption, i.e., $E = \max\{\varepsilon(Q) - A, 0\}$, can lead to an efficient allocation requiring full abatement (see Sans et al. [31] for a discussion).

3.2. The first-best allocation

Under these assumptions, a first-best allocation is given by:

$$\left(Q^{opt}, A^{opt}\right) \in \arg\max_{Q,A\geq 0} \int_0^Q P\left(q\right)dq - c\left(Q\right) - D\left(\max\left\{\varepsilon\left(Q\right) - A, 0\right\}\right) - \kappa(A) \quad (1)$$

This is typically a non-smooth optimization problem, but remember that we have assumed that $D(0) = 0$ and $D'(0) = 0$. The first equality ensures that the optimal level of abatement cannot be greater than emissions because abatement is costly, hence $\varepsilon\left(Q^{opt}\right) - A^{opt} \geq 0$, while the second combined with the positivity of the marginal cost of abatement ensures that this inequality holds strictly. Consequently, the first-best allocation is characterized by the usual first order conditions:

$$P(Q^{opt}) - c'\left(Q^{opt}\right) - D'\left(\varepsilon\left(Q^{opt}\right) - A^{opt}\right) \times \varepsilon'\left(Q^{opt}\right) = 0 \quad (2a)$$

$$D'\left(\varepsilon\left(Q^{opt}\right) - A^{opt}\right) - \kappa'(A^{opt}) = 0 \quad (2b)$$

Let us now introduce the function $\beta(Q) = \frac{P(Q)-c'(Q)}{\varepsilon'(Q)}$ defined on $[0, Q_{\max}]$ where $Q_{\max}$ stands for the optimal level of production without taking into account environmental damage (i.e., $P(Q_{\max}) = c'\left(Q_{\max}\right)$). This function measures, for each $Q \leq Q_{\max}$, the marginal benefit from an additional unit of pollution. Therefore an optimal allocation has the property that the marginal benefit of pollution is equal to (i) the marginal damage and (ii) the marginal cost of abating an additional unit of pollution:

$$\beta(Q^{opt}) = D'\left(\varepsilon\left(Q^{opt}\right) - A^{opt}\right) = \kappa'(A^{opt}) \quad (3)$$

For later use, let us also note that this marginal benefit is decreasing and $\beta(Q_{\max}) = 0$ so that $\beta^{-1} : [0, +\infty] \to [0, Q_{\max}]$ is defined.

4. Upstream monopoly power and first-best regulation

In order to illustrate our main point in the simplest way, we detail the most straightforward case : a representative polluting firm and an upstream monopoly. Three steps are required to show that a policy maker reaches the efficient allocation with a standard Pigouvian tax scheme. We first compute the inverse demand for abatement services under a downstream market clearing assumption. In the second step, we characterize the behavior of the upstream monopolist whatever the Pigouvian tax. In the last step, we show that a Pigouvian tax equal to the marginal damage from the first-best emission level regulates both environmental and market power inefficiencies.

4.1. The demand for abatement services

Since the dirty firm is competitive, we know that its demand for abatement services comes from cost-minimizing behavior which reduces the burden of the environmental constraints. If τ denotes the Pigouvian tax, the smallest cost that this firm is ready to pay conditional on a production level, Q, is given by:

$$C_A(p_A, \tau, Q) = \min_{A\geq 0}\left\{p_A \times A + \tau \times E(Q, A)\right\} \quad (4)$$

Bearing in mind that this activity is completely outsourced, meaning that $E(Q, A) = \max\{\varepsilon(Q) - A, 0\}$, an examination of this cost minimization program shows that the conditional demand for abatement services never exceeds $\varepsilon(Q)$ otherwise the firm purchases unnecessary abatement services and that the objective function is linear in A on $[0, \varepsilon(Q)]$. Both properties imply that this conditional demand is either 0 or $\varepsilon(Q)$ when $p_A > \tau$ or $p_A < \tau$ respectively and any quantity within $[0, \varepsilon(Q)]$ if $p_A = \tau$. So this optimal tax avoidance strategy leads to a minimal additional operating cost of $C_A(p_A, \tau, Q) = \min\{p_A, \tau\} \times \varepsilon(Q)$. The "full production cost" of this firm is therefore given by $c(Q) + C_A(p_A, \tau, Q)$. It includes the cost involved in the tax avoidance choice.

However, to move from conditional to real demand for abatement, we also need to know production level. Since this firm is competitive, its supply simply equates the price of the final good to this full marginal cost,. More precisely, we have:

$$p_Q = c'(Q) + \min\{p_A, \tau\} \times \varepsilon'(Q) \tag{5}$$

If we now introduce the market clearing condition for the final good, we can replace p_Q by $P(Q)$, and, using the previous definition of $\beta(Q)$, i.e., the marginal benefit of an additional unit of pollution, the preceding equation becomes:

$$\beta(Q) := \tfrac{P(Q) - c'(Q)}{\varepsilon'(Q)} = \min\{p_A, \tau\} \Rightarrow Q(p_A, \tau) = \beta^{-1}(\min\{p_A, \tau\}) \tag{6}$$

where $Q(p_A, \tau)$ describes the downstream equilibrium production level for any abatement price and any tax. By using, say, a non-smooth version of Shephard's lemma, we finally deduce the demand for abatement services that the downstream firm outsources at equilibrium. This quantity is of:

$$A^d(p_A, \tau) = \begin{cases} 0 & \text{if } p_A > \tau \\ [0, \varepsilon(\beta^{-1}(\tau))] & \text{if } p_A = \tau \\ \varepsilon(\beta^{-1}(p_A)) & \text{if } p_A < \tau \end{cases} \tag{7}$$

This last equation conveys most of the intuition behind this paper. Since the dirty firm completely outsources clean-up,there is typically no demand for abatement if the price, p_A, proposed by the eco-industry is higher than the Pigouvian tax. In the opposite case, the polluter abates the total amount of pollution generated by its activity, $\varepsilon(\beta^{-1}(p_A))$. This quantity is endogenous since the downstream equilibrium production level $\beta^{-1}(p_A)$ equates price to full marginal cost and therefore incorporates the abatement price. Finally, if the abatement price is equal to the Pigouvian tax, the polluter becomes indifferent between levels of abatement lower than the emissions generated by the equilibrium production level. This means that the elasticity of the demand becomes infinite at $p_A = \tau$. The regulator, by controlling this tax rate, is therefore able to create situations resembling perfect competition, and thus, to implement the first-best regulatory solution. However to verify this point, let us first analyze the monopoly provision of environmental services under alternative tax rates.

4.2. The monopoly provision of environmental services

To construct this quantity, especially if we plan to introduce Cournot competition, it is more convenient to work with the inverse demand. This is quite easy to define. From Eq. (7), the price never exceeds the Pigouvian tax. In the opposite case, $p(A) = \beta\left(\varepsilon^{-1}(A)\right)$, a decreasing function which becomes zero for $A \geq \varepsilon(Q_{\max})$ where $Q_{\max}$ stands for the production level without environmental regulation. This inverse demand is therefore given for all $A \in [0, \varepsilon(Q_{\max})]$ by $\min\{\tau, p(A)\}$ and the monopoly provision of environmental services solves:

$$\max_{A \in [0, \varepsilon(Q_{\max})]} \{\min\{\tau, p(A)\} \times A - \kappa(A)\} \tag{8}$$

The existence of a solution is not a real issue here, since we are maximizing a continuous function on a compact set. But taking the discussion a step further, let us assume - as usual for a monopoly - that the elasticity of $p(A)$ given by $e_p(A) = \frac{p'(A)A}{p(A)}$ is decreasing and $\lim_{A \to \varepsilon(Q_{\max})} e_p(A)$ is larger than -1.[6]. One of the implications of this last assumption is that the monopoly problem without the upper bound on price has a unique solution, A_m. But this also means that for any tax rate $\tau \geq \tau_m = p(A_m)$, the monopoly always provides A_m units of abatement services, i.e $A^m(\tau) = A_m$. In other words, we end up with full pollution abatement at the monopoly price. If the tax rate falls below τ_m, the monopoly is no longer able to charge this price without losing the market. It therefore has an incentive to select the highest price $p_A = \tau$ and to sell the largest quantity of abatement services $A_m(\tau) = p^{-1}(\tau)$ as long as its marginal production cost is lower than the selling price. There is therefore another threshold $\tau_c < \tau_m$ at which the behavior of the monopoly changes again and which is given by:

$$\kappa'(p^{-1}(\tau_c)) = \tau_c \tag{9}$$

Bearing in mind that $p^{-1}(\tau)$ is the quantity that fully abates the downstream equilibrium pollution at price $p_A = \tau$, this new threshold, τ_c, has two readings. It is obviously the lowest tax rate at which the monopoly has an incentive to charge $p_A = \tau$ and to reduce all the downstream pollution. But it is also the competitive price of the abatement services that would prevail if all pollution were reduced. As a consequence, if the tax rate falls below τ_c, even if the monopoly quotes the highest possible price $p_A = \tau$, it has no incentive to deliver the largest quantity of abatement. In fact, from the early definition of the demand curve (see Eq. (7)), we know that for $p_A = \tau$ any quantity $A \in [0, p^{-1}(\tau)]$ belongs to the demand curve because the polluter is indifferent between paying the tax or buying abatement services. This means that it is in the interests of the monopoly to provide abatement services until the price it quotes is equal to its marginal cost, i.e. $A_m(\tau) = (\kappa')^{-1}(\tau)$. But this also means that for tax rates lower than τ_c, it behaves like a competitive firm. From this informal discussion, we conclude that:

[6]Of course, the reader may object that these assumptions are not set on the primary data, especially given that $p(A) = \beta\left(\varepsilon^{-1}(A)\right)$. Other sufficient conditions can be introduced, such as $2e_\varepsilon + e_{\beta'} - e_{\varepsilon'} > 0$ and $e_\varepsilon + e_\beta > 0$, where e denotes the elasticity.

Lemma 1. *Under our assumptions, (i) the monopoly problem (Eq. 8) has a unique solution for each tax rate, which is given by the continuous function:*

$$A^m(\tau) = \begin{cases} (\kappa')^{-1}(\tau) & \text{if } \tau < \tau_c \\ p^{-1}(\tau) = \varepsilon\left(\beta^{-1}(\tau)\right) & \text{if } \tau \in [\tau_c, \tau_m] \\ A_m = p^{-1}(\tau_m) = \varepsilon\left(\beta^{-1}(\tau_m)\right) & \text{if } \tau > \tau_m \end{cases} \tag{10}$$

(ii) the price of these services is $P_A^m(\tau) = \min\{\tau, \tau_m\}$*, and (iii) from Eq. (6), the equilibrium production of the dirty good is:*

$$Q^m(\tau) = \beta^{-1}\left(\min\{\tau, \tau_m\}\right) \tag{11}$$

4.3. The efficient regulation of emissions

With the previous lemma, the die is cast. If the first-best Pigouvian tax, τ^{opt}, which corresponds to the marginal damage of the first-best pollution level, $D'\left(\varepsilon\left(Q^{opt}\right) - A^{opt}\right)$, is lower than τ^c, the regulator is able to implement the first-best allocation, since the downstream market is competitive and the monopoly behaves like a pure competitor. By lemma 1, we have:

$$\forall \tau < \tau_c, \quad P_A^m(\tau) = \kappa'\left(A^m(\tau)\right) \tag{12}$$

It simply remains to ensure that $\tau^{opt} < \tau_c$. So let us assume the contrary. Since at the first-best (see Eq. (3)), $\tau^{opt} = \beta(Q^{opt})$ and β decreasing, we can first claim that the emissions before abatement are lower at the first-best than those generated by the monopoly outcome at a tax rate $\tau = \tau_c$. More precisely:

$$\tau^{opt} \geq \tau_c \Leftrightarrow \underbrace{\varepsilon\left(Q^{opt}\right) = \varepsilon\left(\beta^{-1}\left(\tau^{opt}\right)\right)}_{\text{from Eq. (3)}} \leq \underbrace{\varepsilon\left(\beta^{-1}(\tau_c)\right) = \varepsilon\left(Q^m(\tau_c)\right)}_{\text{from Eq. (11)}} \tag{13}$$

Secondly, if $\tau^{opt} \geq \tau_c$, we can also say, from the definition of τ_c (see Eq. (9)), that the first-best abatement level is larger than the abatement required to fully reduce the emissions generated by the monopoly equilibrium for $\tau = \tau_c$. In fact:

$$\tau^{opt} \geq \tau_c \Leftrightarrow \underbrace{A^{opt} = (\kappa')^{-1}\left(\tau^{opt}\right)}_{\text{from Eq. (3)}} \geq \underbrace{p^{-1}(\tau_c) = \varepsilon\left(\beta^{-1}(\tau_c)\right) = \varepsilon\left(Q^m(\tau_c)\right)}_{\text{from Eqs.(9).and.(10)}} \tag{14}$$

Both observations imply that the optimal abatement level is higher than the amount of pollution generated by the optimal production level, i.e. $A^{opt} \geq \varepsilon\left(Q^{opt}\right)$. But we have assumed that $D'(0) = 0$, so we know from our early discussion of the optimal outcome (section 3.2) that $A^{opt} < \varepsilon\left(Q^{opt}\right)$. From this contradiction, we can therefore say:

Proposition 1. *Even if an upstream monopoly controls the price of the environmental services while the downstream commodity market remains competitive, the regulator reaches the first-best outcome by setting the Pigouvian tax at the marginal damage of the emissions (evaluated at the first-best), i.e., by setting* $\tau^{opt} = D'\left(\varepsilon\left(Q^{opt}\right) - A^{opt}\right)$*.*

The reader may perhaps object that it is too restrictive to say that there is no marginal damage at a zero-emission level. This assumption, for instance, excludes the case of constant marginal damage. The next section gives several extensions and relaxes this restriction.

5. Some extensions

Our first extension deals with the above-mentioned issue. But, we can also examine whether the result holds when the regulator uses a different incentive-based mechanism, such as tradeable pollution permits. The answer is again yes, as long as this new market is competitive. We then relax the representative polluting firm assumption by introducing heterogeneous dirty firms both in terms of their production costs and their emissions. We finally consider the case of upstream Cournot competition between heterogeneous eco-service providers.

5.1. Efficient regulation and full abatement

To illustrate this point, let us return to the construction of the efficient outcome and relax $D'(0) = 0$. This outcome solves the optimization program (Eq. (1)) introduced in Section 3.2. But if we only assume that $D(0) = 0$, we can only argue that $\varepsilon(Q) - A \geq 0$, (i.e., without strict inequality). The interior first-order optimality conditions given by Eqs. (2a) and (2b) must therefore be amended. If λ denotes the Lagrangian multiplier associated with this full abatement constraint, the new FOCs become:

$$\begin{cases} P(Q^{opt}) - c'\left(Q^{opt}\right) - \left(D'\left(\varepsilon\left(Q^{opt}\right) - A^{opt}\right) - \lambda\right)\varepsilon'\left(Q^{opt}\right) = 0 \\ D'\left(\varepsilon\left(Q^{opt}\right) - A^{opt}\right) - \kappa'(A^{opt}) - \lambda = 0 \\ \lambda\left(\varepsilon\left(Q^{opt}\right) - A^{opt}\right) = 0 \text{ and } \lambda \geq 0 \end{cases} \tag{15}$$

If this constraint is not binding, we are back, of course, to the case of partial abatement situation analyzed above. So let us concentrate on the case in which $\lambda > 0$, meaning that there is full abatement since $\varepsilon\left(Q^{opt}\right) = A^{opt}$. In this situation, the first and second equations of system (15) suggest that, for an efficient allocation, the marginal benefit $\beta(Q^{opt})$ of an additional unit of pollution must be equal to the marginal abatement cost. But to achieve full abatement, this marginal benefit only needs to be smaller than the marginal damage from the first unit of pollution. This situation essentially occurs if $D'(0)$ is high enough. In this case, the efficient allocation verifies:

$$E^{opt} = \varepsilon\left(Q^{opt}\right) - A^{opt} = 0 \tag{16a}$$

$$\beta(Q^{opt}) = \kappa'(A^{opt}) < D'(0) \tag{16b}$$

instead of the interior condition introduced in Eq. (3).

Let us now return to the monopoly case. Since marginal damage is not part of the definition of the different behaviors, the monopoly outcome depicted in Lemma 1 remains unchanged. So, if we want to extend our result, we simply need to set a tax rate such that the monopoly outcome at this tax rate satisfies conditions (16a) and (16b). Let us take $\tau = \tau^c$. From Lemma 1, the equilibrium abatement and production levels are $A^m(\tau_c) = \varepsilon\left(\beta^{-1}\left(\tau_c\right)\right)$ and $Q^m(\tau_c) = \beta^{-1}\left(\tau_c\right)$, so that condition (16a) is immediately satisfied. Moreover since τ_c verifies $\tau_c = \kappa'(\varepsilon\left(\beta^{-1}\left(\tau_c\right)\right))$, we also have that $\beta\left(Q^m(\tau_c)\right) = \kappa'\left(A^m(\tau_c)\right)$. It should, however, be noted that in order to obtain a first-best outcome requiring full abatement, the tax rate needs to be strictly lower than the marginal damage, i.e. $\tau_c < D'(0)$. The intuition behind this last observation directly follows from Lemma 1: if the tax rate is higher than τ_c, the monopoly outcome induces full pollution abatement but at a lower level of production of both final goods and abatement services. We can therefore state that:

Proposition 2. *Assume that the marginal damage of the first unit of pollution is high enough for full abatement to become the efficient outcome. If the regulator sets the Pigouvian tax at $\tau^{opt} = \tau_c$ given by Eq. (9), he again obtains the first-best outcome. Moreover this tax rate τ_c is, in this case, lower than the marginal damage.*

5.2. Pollution permit market

Let us now verify that our result also holds if the regulator implements a pollution permit market instead of a Pigouvian tax. To illustrate this point, let us return to the monopoly case depicted in Section 3 and introduce a competitive market of pollution permits. The regulator sets the pollution cap $\bar{E}$. Without loss of generality we assume that pollution permits are sold at auction.[7] One permit corresponds to one unit of emission and the competitive price of these permits is denoted by p_E.

At the agent level, the current permit price operates like an environmental tax. Under our assumptions, the results obtained in Section 3 concerning the inverse demand and the supply of abatement services by the monopoly extend to this case: it simply remains for us to replace the Pigouvian tax τ by the price p_E of the emission permits. If we go back to Lemma 1 the quantities $A^m(p_E)$ and $Q^m(p_E)$ in Eqs. (10) and (11) now turn out to be the equilibrium abatement and production levels conditional on a pollution permit price p_E. From their definition, we can say that the demand for pollution permits is given by:

$$E^D(p_E) = \begin{cases} \varepsilon\left(\beta^{-1}(p_E)\right) - (\kappa')^{-1}(p_E) \text{ if } p_E < \tau_c \\ 0 \text{ if } p_E \geq \tau_c \end{cases} \tag{17}$$

Now let us assume the pollution cap chosen by the regulator is $\bar{E} = \varepsilon(Q^{opt}) - A^{opt} > 0$, meaning that we implicitly assume that $D'(0) = 0$. From the definition of the first-best (see Eqs. (2a), (2b)), it is immediately seen that $\tau^{opt} = D'(\varepsilon(Q^{opt}) - A^{opt})$, which is strictly lower than τ_c (see our discussion in Section 4.3), is an equilibrium price for the pollution permit market, i.e. $E^D(\tau^{opt}) = \bar{E}$. Moreover we observe that the demand for pollution permits is decreasing for all $p_E < \tau_c$:

$$\frac{dE^D(p_E)}{dp_E} = \frac{\varepsilon'\left(\beta^{-1}(p_E)\right)}{\beta'\left(\beta^{-1}(p_E)\right)} - \left(\kappa''\left((\kappa')^{-1}(p_E)\right)\right)^{-1} < 0 \tag{18}$$

since, under our assumptions, $\kappa'', \varepsilon' > 0$ and $\beta' < 0$. This implies that the price $p_E = \tau^{opt}$ is the unique equilibrium of the market when the pollution cap is $\bar{E}$. In other words,

[7] For simplicity, we do not introduce the initial distribution of pollution permits explicitly. Following Montgomery [21], the competitive equilibrium of a pollution permit market is obtained irrespective of the initial distribution of permits.

when the regulator sells an amount $\bar{E}$ he knows for sure that the unique equilibrium price implements the first-best allocation.

However, it should be noted that this argument only partially extends to the case where $D'(0) > 0$. It only works if the full abatement constraint (see Eq. (15)) is not binding at the first-best.[8] In fact if full abatement is required, there is no point in organizing a pollution permit market because there is nothing to trade. To conclude this discussion, we can therefore say:

Proposition 3. *If pollution is regulated by a pollution permit market, the regulator also achieves an efficient allocation by choosing the optimal pollution cap given by $\bar{E} = \varepsilon\left(Q^{opt}\right) - A^{opt}$. This however requires that $\bar{E} > 0$.*

5.3. Heterogeneous polluters.

It is also interesting to see whether this result extends to heterogeneous polluters. Let us introduce m polluting firms, indexed by j, with different cost and emission functions, $c_j(q)$ and $\varepsilon_j(q)$, each of them satisfying the assumptions in Section 3.1. All the other assumptions are maintained, especially those concerning the marginal damage at 0, so that an efficient allocation is now given by:

$$\forall j \quad P(\textstyle\sum_{j=1}^{m} q_j^{opt}) - c_j'\left(q_j^{opt}\right) - D'\left(\sum_{j=1}^{m}\varepsilon_j\left(q_j^{opt}\right) - A^{opt}\right).\varepsilon_j'\left(q_j^{opt}\right) = 0 \tag{19a}$$

$$D'\left(\textstyle\sum_{j=1}^{m}\varepsilon_j\left(q_j^{opt}\right) - A^{opt}\right) - \kappa'(A^{opt}) = 0 \tag{19b}$$

Let us now turn to the market outcome. The intuition behind this extension is quite simple. Even if the polluting firms are heterogeneous in costs and emissions, they invariably choose their level of abatement by comparing the price p_A with the Pigouvian tax τ. We can therefore expect the *aggregate* demand for abatement goods to behave in the same way: no abatement if $p_A > \tau$, full abatement denoted $A_f(p_A)$ if $p_A < \tau$ and any situation between the two if $p_A = \tau$. Moreover, if the demand on the domain corresponding to full abatement is still decreasing and bounded from above, the inverse demand has the same structure as that obtained in Section 4.1. So, with similar assumptions on its elasticity, the properties of the monopoly outcome provided in Lemma 1 should extend to the case of heterogeneous polluters.

The main weakness of this argument is that computing the aggregate level of abatement corresponding to full pollution reduction ($A_f(p_A)$) and, more generally, constructing the market-clearing production levels for all τ and p_A, now become intricate operations. In fact - as in Section 4.1 - it is easy to compute the individual conditional demand for abatement services and the cost function related to this activity. But to compute the market-clearing production level, we now face a system of m equations, since for each

[8] In fact,. from our discussion in section 5.1, we can say that the full abatement constraint is not binding (even weakly) if $D'(0) > \tau_c$.

firm, the price of the polluting good is equal to the full marginal cost including abatement cost. In other words, these individual production levels solve:

$$\forall j = 1, \ldots, m \quad P\left(\sum_{j=1}^{m} q_j\right) = c_j'(q_j) + \min\{p_A, \tau\} \times \varepsilon_j'(q_j) \tag{20}$$

instead of the single equation given by Eq. (6). Nevertheless, it can be shown that:

Lemma 2. *Under our assumptions on demand, costs and emissions, the system of Eqs. (20) admits a unique solution* $(q_j(\min\{p_A, \tau\}))_{j=1}^{m}$. *Moreover,* $A_f(p_A) = \sum_{j=1}^{m} \varepsilon_j(q_j(p_A))$ *- the total quantity of abatement good which induces full pollution reduction - is decreasing (for all* $p_A \leq \tau$*) and bounded from above by* $A_{\max} = \sum_{j=1}^{m} \varepsilon_j(q_j(0))$.

It finally remains to verify that the Pigouvian tax $\tau^{opt} = D'\left(\sum_{j=1}^{m} \varepsilon_j\left(q_j^{opt}\right) - A^{opt}\right)$ (i) can achieve the first-best outcome and (ii) is lower than the highest tax, now given by $\tau_c' = \kappa'(A_f(\tau_c'))$,[9] that induces competitive behavior by the abatement producer. If the second point is satisfied, the first is easily obtained by identification. In fact, if $\tau^{opt} < \tau_c'$, the eco-service firm equates its marginal cost to the Pigouvian tax, i.e., $\tau^{opt} = \kappa'(A)$, so that condition (19b) is satisfied. Moreover, since in this case the price of the abatement good is τ^{opt}, it follows that the set of Eqs (20), describing the equilibrium production levels, corresponds exactly to the efficiency conditions given by Eqs. (19a). It remains to verify that $\tau^{opt} < \tau_c'$. The argument works as in section 4.3. If $\tau^{opt} \geq \tau_c'$, we can now claim by the definition of τ_c' and by $A_f'(p_A) \leq 0$, that abatement activity still exceeds the total amount of emissions, i.e. $(\kappa')^{-1}(\tau^{opt}) \geq \sum_{j=1}^{m} \varepsilon_j(q_j(\tau^{opt}))$. Seeing that these quantities are the efficient ones, this rules out the residual pollution implied by the assumption that $D'(0) = 0$.[10] We can therefore state that:

Proposition 4. *Even if the polluting sector is composed of firms heterogeneous in costs and emissions, the regulator can neutralize the monopoly power on the abatement service market and obtain the first-best solution by setting the tax rate at the marginal damage evaluated at the first-best.*

5.4. Cournot competition in the eco-industry

Let us now restore the representative polluting firm assumption, and consider the case of upstream Cournot competition by introducing n heterogeneous firms, indexed by i, into the eco-industry, each characterized by a specific cost function $\kappa_i(a)$. Given that all the other assumptions are maintained, an efficient allocation now verifies:

$$\forall i = 1 \ldots, n \quad \beta(Q^{opt}) = D'\left(\varepsilon\left(Q^{opt}\right) - \sum_{i=1}^{n} a_i^{opt}\right) = \kappa_i'\left(a_i^{opt}\right) \tag{21}$$

[9] By a similar argument to that in the proof of Lemma 1, it can be shown that τ_c' exists and is unique.

[10] This argument can, as in Section 5.1, be extended to the case $D'(0) > 0$. The computation of the first-best is nevertheless more tedious. Due to downstream heterogeneity in emissions, some firms abate all their emissions while others not. By in any case the result is obtained by setting $\tau = \tau_c'$.

Does a Pigouvian tax of $\tau^{opt} = D'\left(\varepsilon\left(Q^{opt}\right) - \sum_{i=1}^{n} a^{opt}\right)$ still lead to the first-best allocation?

To answer this question, let us directly examine the best responses of these Cournot players, since the behavior of the polluting firm remains unchanged by construction (see Section 4.1). If we denote by $A_{-i} = \sum_{j=1,j\neq i}^{n} a_j^{opt}$ the aggregated abatement supply of the opponents, the best response of Firm i is given by:

$$BR_i(A_{-i}, \tau) \in \arg\max_{a_i} \left\{\min\left\{\tau, p(a_i + A_{-i})\right\} \times a_i - \kappa_i\left(a_i\right)\right\} \tag{22}$$

where $p(A) = \beta\left(\varepsilon^{-1}(A)\right)$ stands, as usual, for the inverse demand corresponding to full abatement behavior by the polluting firm.

The main difference here from the monopoly case stems from the fact that the different kinds of behaviors identified in Lemma 1 now depend, at a given tax rate, on the behavior of the other players. First, it is quite obvious that a firm is now able to exert its full market power only if its best response against the choices of the others, A_{-i}, is such that the price remains lower than the tax rate. In other words, if we denote by $br_i(A_{-i})$ the best response of i when the price is not bounded by the tax rate, we can identify a first threshold $A_{-i}^m(\tau)$ implicitly given by:

$$p(br(A_{-i}^m(\tau)) + A_{-i}^m(\tau)) = \tau \tag{23}$$

and if $A_{-i} \geq A_{-i}^m(\tau)$, we can say that Firm i exerts full market power and plays $BR_i(A_{-i}, \tau) = br_i(A_{-i})$. Now let us assume that A_{-i} falls below $A_{-i}^m(\tau)$. In this case Firm i cannot reach its best solution, since the upper bound on price matters. As in the monopoly case, but now given the strategies of the others, Firm i provides the largest quantity of services at the highest price, τ. In other words, it sells $BR_i(A_{-i}, \tau) = p^{-1}(\tau) - A_{-i}$. But as A_{-i} decreases, Firm i's production increases. It can therefore, as in the monopoly case, end up with its own marginal production cost being higher than the constant price τ. This gives us the opportunity to identify a second threshold $A_{-i}^c(\tau)$ given by:

$$\kappa_i'\left(p^{-1}(\tau) - A_{-i}^c(\tau)\right) = \tau \Leftrightarrow A_{-i}^c(\tau) = p^{-1}(\tau) - \left(\kappa_i'\right)^{-1}(\tau) \tag{24}$$

such that for $A_{-i} < A_{-i}^c(\tau)$, Firm i behaves like a pure competitor by equating its marginal cost to the market price.

Of course, this informal argument only works if both thresholds, $A_{-i}^c(\tau) < A_{-i}^m(\tau))$, are strictly positive, otherwise some of these cases are vacuous. A formal construction of the best response is provided in the appendix. We even show that these three situations only occur when the rate τ is lower than τ_c^i given by $\tau_c^i = \kappa_i'(p^{-1}(\tau_c^i))$. But, since we are concerned purely with situations in which eco-industry competitive behaviors arise, we only spell out the characterization of the best response for $\tau \leq \tau_c^i$.

Lemma 3. *Under the assumption on the elasticity of $p(A)$ introduced in Section 4.2, the best response of an eco-service firm, for any tax rate $\tau < \tau_c^i$ and any $A_{-i} \in [0, \varepsilon^{-1}(Q_{\max})]$, is given by:*

$$BR(A_{-i}, \tau) = \begin{cases} \left(\kappa_i'\right)^{-1}(\tau) & \text{if } A_{-i} < A_{-i}^c(\tau) \\ p^{-1}(\tau) - A_{-i} & \text{if } A_{-i}^c(\tau) \leq A_{-i} \leq A_{-i}^m(\tau)) \\ br_i(A_{-i}) & \text{if } A_{-i} \geq A_{-i}^m(\tau)) \end{cases} \tag{25}$$

Moreover, this best response is continuous and non-increasing with A_{-i}.

This last lemma also tells us that competitive behavior is part of abater i's best response only if the tax rate is lower than τ_c^i. So let us concentrate in the rest of the argument on taxes lower than $\tau_c^{\min} = \min_{i=1,\ldots,n} \{\tau_c^i\}$ so that the best response of each player incorporates competition. In this case, it can be shown that:

Lemma 4. *For any tax $\tau < \tau_c^{\min}$, the unique Cournot equilibrium leads, for each firm, to a supply of eco-services of $a_i^C(\tau) = (\kappa_i')^{-1}(\tau)$, while the price of these services is $P_A^C(\tau) = \tau$. From Eq. (6), we observe that the production of the dirty good is $Q^C(\tau) = \beta^{-1}(\tau)$.*

In other words, for any $\tau < \tau_c^{\min}$ and even if there is Cournot competition in the eco-industry, the unique equilibrium is such that the firms in the eco-industry behave like pure competitors by equating their marginal cost to the price of the abatement services. Thus, to confirm that the regulator is able to implement the first-best, we need to verify that the first-best tax rate $\tau^{opt} = D'\left(\varepsilon\left(Q^{opt}\right) - \sum_{i=1}^{n} a_i^{opt}\right)$ is lower than $\tau_c^{\min}$. If this is not the case, there exists at least one agent, say i_0, for which $\tau^{opt} > \kappa_{i_0}'(p^{-1}(\tau^{opt}))$. But this implies, for our characterization of an optimal allocation (Eq. (21)), that:

$$a_{i_0}^{opt} = \left(\kappa_{i_0}'\right)^{-1}\left(\tau^{opt}\right) > p^{-1}\left(\tau^{opt}\right) = \varepsilon\left(\beta^{-1}\left(\tau^{opt}\right)\right) = \varepsilon(Q^{opt}) \tag{26}$$

so that $\sum_{i=1}^{n} a_{i_0}^{opt} > \varepsilon(Q^{opt})$, since all the $a_i^{opt} \geq 0$. In other words there is, at the optimum, more abatement than emissions, which is a contradiction. We can therefore say:

Proposition 5. *If there is Cournot competition in an eco-service industry and pure competition in the polluting sector, the first-best allocation can be reached by setting the tax rate to the marginal damage.*

6. Concluding remarks

The EGSS is highly concentrated and the economic literature has so far mainly analyzed how this feature impacts the design of environmental regulations. However, no study has yet analyzed the extent to which distinguishing between abatement goods and abatement services matters for environmental regulation. By an abatement service, we mean an activity which is totally outsourced by the polluting firm to a specialized firm and is purchased on a market at a given price per unit of abatement. By contrast, what we term an abatement good is used directly by the polluting firm. This distinction is of particular interest for policy makers. There are two market failures in our economy: market power on the abatement service market and pollution generated by downstream firms. Yet our results suggest that the regulator can reach the first-best outcome with only one tool: environmental regulation.

The main intuition behind our results is based on the idea that a polluter, when he outsources abatement services, only makes a trade-off between the price of this service and the cost of non-compliance with an environmental policy. In other words, he does not care about the way the pollution is reduced by the eco-service firm. This simple observation drastically modifies his purchasing behavior. In fact, we have shown that the demand for abatement becomes infinitely elastic over a certain range. This at least partially offsets upstream market power, meaning that if the Pigouvian tax is set at the right level, the monopoly will choose the first-best level of production. We explored this argument in the context of a downstream representative polluting firm and an upstream monopoly. We then extended our model to check the robustness of the result. We first set assumptions, such as that total abatement is allowed. We secondly considered a pollution permit market instead the Pigouvian tax, and thirdly, introduced heterogenous polluters. Finally we extended our results to upstream Cournot competition between heterogenous abaters.

Our findings nevertheless have several limitations. In this paper, we essentially explore the case of upstream market power. However, if we add other market failures or if we relax the benevolent regulator assumption, the results clearly change.

Concerning market failures, it is first obvious that the introduction of downstream imperfect competition between polluters challenges our results. In this case the regulator, even if he is able to restore upstream perfect competition, can be expected to have an incentive to lower the Pigouvian tax to below its first-best level in order to limit the reduction in production of the final good induced by this additional market failure. Secondly, our results largely rely on the assumption that the upstream eco-firms propose linear price contracts on an anonymous spot market. If the abater has the option of proposing non-linear price schedules, for instance, he will be able to "smooth" the polluter tax avoidance strategy. Finally, if a pollution permit market is organized, a polluting firm may exert a dominant position, i.e., a simple manipulation (see Hahn [18] and Westskog [39]) or by manipulating the costs of its opponents on the output market, i.e. an exclusionary manipulation (see Misiolek and Elder [20], Sartzetakis [32] or Von der Fehr [38]).

In this paper, we also restrict our attention to a benevolent regulator controlling a closed economy. However, lobbies are recognized to influence the definition of environmental policy (Aidt [1]), and abatement services are often exchanged on an international market. Canton [3] studies the role of lobbies in the case of an eco-industry providing environmental goods. Moreover, in an open economy, each firm is subject to national environmental regulations. In this case, environmental policies can be used in a strategic way (see for instance Barrett [2] or Hamilton and Requate [17]). Nimubona [22] studies the effect on the eco-industry sector of reductions in trade barriers that were agreed in the Doha Round of the WTO.

Further research could usefully investigate whether taking into account these additional features would challenge or alter our results.

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Appendix A. Proof of Lemma 1

We need to solve:

$$\max_{A\in[0,\varepsilon(Q_{\max})]} \underbrace{\left[\min\left\{\tau, p\left(A\right)\right\} A - \kappa\left(A\right)\right]}_{\pi(A,\tau)} \text{ where } p\left(A\right) = \beta\left(\varepsilon^{-1}\left(A\right)\right) \tag{A.1}$$

Step 1: Existence of a unique solution

Since we maximize $\pi(A,\tau)$ over a compact set, it remains to verify that $\pi(A,\tau)$ is strictly concave in A. Moreover, $\kappa\left(A\right)$ being strictly convex, it remains to check that $\min\left\{\tau A, p\left(A\right)A\right\}$ is concave. But let us first observe that, under the assumption that $e_p(\varepsilon(Q_{\max})) > -1$ and $\frac{de_p(A)}{dA} \leq 0$, $(p(A)A)$ is concave since :

$$\frac{d^2}{(dA)^2}(p(A)A) = p'(A)(e_p(A)+1) + p(A)\frac{de_p(A)}{dA} \leq 0 \tag{A.2}$$

It follows that $\forall \lambda \in [0,1]$ and $\forall A_1, A_2 \in [0, \varepsilon(Q_{\max})]$

$$\begin{aligned} &\min\{\tau(\lambda A_1 + (1-\lambda)A_2), p((\lambda A_1 + (1-\lambda)A_2))(\lambda A_1 + (1-\lambda)A_2)\} \\ &\geq \min\{\lambda \tau A_1 + (1-\lambda)\tau A_2, \lambda p(A_1)A_1 + (1-\lambda)p(A_2)A_2\} \text{ (concavity of } p(A)A) \\ &\geq \lambda \min\{\tau A_1, p(A_1)A_1\} + (1-\lambda)\min\{\tau A_2, p(A_2)A_2\} \text{ (concavity of the } \min\{x,y\}\text{)} \end{aligned} \tag{A.3}$$

Step 2: *Construction of the thresholds*

This program is not smooth but nevertheless concave. This means (see Rockafellar [30]) that an optimum is reached iff $0 \in \partial_A \pi$ where $\partial_A \pi$ denotes the sub-derivative of $\pi(A,\tau)$ with respect to A. By computation, we get:

$$\partial_A \pi = \begin{cases} \tau - \kappa'(A) & \text{if } A < p^{-1}(\tau) \\ \left[\underbrace{\tau + p'(p^{-1}(\tau))p^{-1}(\tau) - \kappa'\left(p^{-1}(\tau)\right)}_{:=\phi_m(\tau)}, \underbrace{\tau - \kappa'\left(p^{-1}(\tau)\right)}_{:=\phi_c(\tau)}\right] & \text{if } A = p^{-1}(\tau) \\ p(A) + p'(A)A - \kappa'(A) & \text{if } A > p^{-1}(\tau) \end{cases} \tag{A.4}$$

Now observe that $\phi_c(\tau) = 0$ and $\phi_m(\tau) = 0$ implicitly defines the two thresholds τ_c and τ_m. It remains to verify that these thresholds exist, are unique and that $\tau_c < \tau_m$. These results directly follow from the next observations:

(i) ϕ_c **and** ϕ_m **are both increasing**. More precisely, $\phi_c'(\tau) = 1 - \frac{\kappa''\left(p^{-1}(\tau)\right)}{p'(p^{-1}(\tau))} > 0$, and

$$\begin{aligned} \phi_m'(\tau) &= \tfrac{d}{d\tau}\left((p(A) + p'(A)A) - \kappa(A)|_{A=p^{-1}(\tau)}\right) \\ &= \underbrace{\left(\tfrac{d^2}{(dA)^2}(p(A)A) - \kappa''(A)\right)}_{<0 \text{ (see Eq. (A.2))}}\Bigg|_{A=p^{-1}(\tau)} \times \underbrace{\tfrac{1}{p'(p^{-1}(\tau))}}_{<0} > 0 \end{aligned} \tag{A.5}$$

(ii) $\phi_c(0) < 0$ **and** $\phi_m(0) < 0$. Let us remember that $p(\varepsilon(Q_{\max})) = 0$ so that $p^{-1}(0) = \varepsilon(Q_{\max})$, it follows that $\phi_c(0) = -\kappa'(\varepsilon(Q_{\max})) < 0$ and $\phi_m(0) = p'(\varepsilon(Q_{\max}))\varepsilon(Q_{\max}) - \kappa'(\varepsilon(Q_{\max})) < 0$

(iii) $\phi_c(\tau_m) > 0$ **and** $\lim_{\tau\to+\infty}\phi_m(\tau) > 0$. The first follows from the fact that $\phi_c(\tau) = \phi_m(\tau) - p'(p^{-1}(\tau))p^{-1}(\tau)$ and $\phi_m(\tau_m) = 0$ so that $\phi_c(\tau_m) = -p'(p^{-1}(\tau_m))p^{-1}(\tau_m) > 0$. Concerning the second result, we note:

$$\lim_{\tau\to+\infty}\phi_m(\tau) = \lim_{\tau\to+\infty}\tau\left(1 + e_p(A)|_{A=p^{-1}(\tau)}\right) - \lim_{\tau\to+\infty}\kappa'\left(p^{-1}(\tau)\right) \tag{A.6}$$

The second term of the r.h.s. is clearly bounded since $p^{-1}(\tau) \in [0, \varepsilon(Q_{\max})]$. If we now remember that $e_p(A)$ is decreasing and $e_p(\varepsilon(Q_{\max})) > -1$, we have $\lim_{\tau\to+\infty}\phi_m(\tau) = +\infty$.

Step 3: *The optimal provision of abatement services*

Let us come back to the subdifferential given by Eq. (A.4). Similar to (i) of Step 2, it can now be argued that the first and the last equation of Eqs. (A.4) are both decreasing functions. Let us also note that (i) $\lim_{A\to0}(\tau - \kappa'(A)) = \tau \geq 0$ and (ii)

$$\begin{aligned} lim_{A\to\varepsilon(Q_{\max})}(p(A) + p'(A)A - \kappa'(A)) &= lim_{A\to\varepsilon(Q_{\max})}(p(A)(1 + e_p(A)) - \kappa'(A)) \\ &= -\kappa'(\varepsilon(Q_{\max})) \text{ (since } p(\varepsilon(Q_{\max})) = 0 \text{ and } e_p(A) \text{ bounded)} \end{aligned} \tag{A.7}$$

From these observations, and since at a maximum $0 \in \partial_A \pi$, we can immediately say that:
(i) if $\phi_c(\tau) < 0$ or, equivalently, $\tau < \tau_c$, the zero of $\partial_A \pi$ is given by $\tau - \kappa'(A) = 0$, so that $A = (\kappa')^{-1}(\tau)$
(ii) if $\phi_m(\tau) \leq 0$ and $\phi_c(\tau) \geq 0$, or $\tau \in [\tau_c, \tau_m]$, the zero is obtained for $A = p^{-1}(\tau)$
(iii) if $\phi_m(\tau) > 0$, *i.e.* $\tau > \tau_m$, the optimal provision solves the last equation and this is nothing more than the standard monopoly solution associated with $p(A)$ (i.e. without the kink introduced by the min function).

Appendix B. Proof of Lemma 2

Step 1: Existence of a solution

Let us denote by $Q = \sum_{j=1}^{m} q_j$ and let us take $\min\{\tau, p_A\}$ as given. We observe that (i) the r.h.s. of each equation of Eqs. (20) is increasing in q_j since $c_j'', \varepsilon_j'' > 0$ (ii) the range of these functions is $[0, +\infty)$ since $c_j'(0) = \varepsilon_j'(0) = 0$ and both functions go to $+\infty$ as $q_j \to +\infty$. We can therefore reverse the function given by the r.h.s. and say that $q_j = \phi_j(Q)$. Moreover, we also observe that (i) $\lim_{Q\to 0} \phi_j(Q) = +\infty$ since $\lim_{Q\to 0} P(Q) = +\infty$ so that the equality (20) requires that $q_j \to +\infty$, and (ii) $\lim_{Q\to +\infty} \phi_j(Q) = 0$ since $\lim_{Q\to +\infty} P(Q) = 0$ and therefore $q_j \to 0$ to maintain the equality.

Let us now aggregate over the q_j. We obtain $Q = \sum_{j=1}^{m} \phi_j(Q)$. So if there exists a solution in Q to this equation our existence problem is solved. It remains to observe that (i) $\Phi(Q) = Q - \sum_{j=1}^{m} \phi_j(Q)$ is continuous (ii) $\lim_{Q\to 0} \Phi(Q) = -\infty$ and (iii) $\lim_{Q\to +\infty} \Phi_j(Q) = +\infty$.

Step 2: Uniqueness of the solution

Let us set $K = \min\{\tau, p_A\}$ and write the system (20) as:

$$\Psi\left((q_j)_{j=1}^{m}, K\right) = \left(c_j'(q_j) + K.\varepsilon_j'(q_j)\right)_{j=1}^{m} - P\left(\sum_{j=1}^{m} q_j\right) e \text{ with } e' = (1, \ldots, 1)$$

By computation, we observe that $\partial_{(q_j)_{j=1}^{m}} \Psi = D - P'\left(\sum_{j=1}^{m} q_j\right) e \cdot e'$ where D is a diagonal matrix whose generic term is $c_j''(q_j) + K\varepsilon_j''(q_j)$. This symmetric matrix is clearly positive definite since c_j'', $\varepsilon_j'' > 0$ and $P' < 0$. It follows from Gale-Nikaido (1965 Theorem 6) that the solution $(q_j(K))_{j=1}^{m}$ of $\Psi\left((q_j)_{j=1}^{m}, K\right) = 0$ is unique for every K.

Step 3: $A_f(K) = \sum_{j=1}^{m} \varepsilon_j(q_j(K))$ is decreasing

Let us first observe from the implicit function theorem applied to $\Psi\left((q_j)_{j=1}^{m}, K\right) = 0$ that $\frac{\partial (q_j)_{j=1}^{m}}{\partial K} = -\left(\partial_{(q_j)_{j=1}^{m}} \Psi\right)^{-1} \cdot \left(\varepsilon_j'(q_j)\right)_{j=1}^{m}$. It follows that:

$$\frac{dA_f}{dK} = \left(\left(\varepsilon_j'(q_j)\right)_{j=1}^{m}\right)' \cdot \frac{\partial (q_j)_{j=1}^{m}}{\partial K} = -\underbrace{\left(\left(\varepsilon_j'(q_j)\right)_{j=1}^{m}\right)' \cdot \left(\partial_{(q_j)_{j=1}^{m}} \Psi\right)^{-1} \cdot \left(\varepsilon_j'(q_j)\right)_{j=1}^{m}}_{>0} < 0$$

since the inverse of a positive definite matrix remains positive definite.

Appendix C. Proof of Lemma 3

We need to solve for all $A_{-i} \in [0, \varepsilon(Q_{\max})]$

$$\max_{a_i \in [0, \varepsilon(Q_{\max}) - A_{-i}]} \underbrace{\left[\min\{\tau, p(a_i + A_{-i})\}(a_i + A_{-i}) - \kappa_i(a_i)\right]}_{\pi_i(a_i, A_{-i}, \tau)} \text{ where } p(A) = \beta\left(\varepsilon^{-1}(A)\right)$$

Step 1: $\pi_i(a_i, A_{-i}, \tau)$ is strictly concave in a_{-i}

Under the assumption that $e_p(\varepsilon(Q_{\max})) > -1$ and $\frac{de_p(A)}{dA} \leq 0$, we have for $a_i > 0$:

$$\begin{aligned} 0 &> \tfrac{a_i}{A}\left(p'(A)\left(e_p(A)+1\right)+p(A)\frac{de_p(A)}{da}\right) = P''(A)a_i + \tfrac{2a_i}{A}P'(A) \\ &> P''(A)a_i + 2P'(A) = \tfrac{\partial^2}{(\partial a_i)^2}\left[p\left(a_i+A_{-i}\right)a_i\right] \end{aligned}$$

We can therefore use the same argument as in Step 1 of Lemma 1 in order to show that $\pi_i(a_i, A_{-i}, \tau)$ is strictly concave in a_i. We simply need to decompose A in $(a_i + A_{-i})$ and take a convex combination of two a_i.

Step 2: *the subdifferential and the thresholds*

Let us now compute the sub-derivate of $\pi_i(a_i, A_{-i}, \tau)$ with respect to a_i. For $A_{-i} < p^{-1}(\tau)$, we obtain:

$$\partial_{a_i}\pi = \begin{cases} \tau - \kappa_i'(a_i) & \text{if } a_i < p^{-1}(\tau) - A_{-i} \\ \left[\varphi_m^i(\tau, A_{-i}), \varphi_c^i(\tau, A_{-i})\right] & \text{if } a_i = p^{-1}(\tau) - A_{-i} \\ \underbrace{p\left(a_i + A_{-i}\right) + p'\left(a_i + A_{-i}\right)a_i - \kappa_i'\left(a_i\right)}_{:=\psi_i(a_i, A_{-i})} & \text{if } a_i > p^{-1}(\tau) - A_{-i} \end{cases} \tag{C.1}$$

$$\text{with } \begin{cases} \varphi_m^i(\tau, A_{-i}) = \tau + p'(p^{-1}(\tau))\left(p^{-1}(\tau) - A_{-i}\right) - \kappa_i'\left(p^{-1}(\tau) - A_{-i}\right) \\ \varphi_c^i(\tau, A_{-i}) = \tau - \kappa_i'\left(p^{-1}(\tau) - A_{-i}\right) \end{cases}$$

If $A_{-i} \geq p^{-1}(\tau)$, the first and even the second line (if $A_{-i} > p^{-1}(\tau)$) are simply vacuous.

So let us for the moment assume that $A_{-i} < p^{-1}(\tau)$ and let us introduce the thresholds $A_{-i}^c(\tau)$ and $A_{-i}^m(\tau)$ given by $\varphi_c^i\left(\tau, A_{-i}^c(\tau)\right) = 0$ and $\varphi_m^i\left(\tau, A_{-i}^m(\tau)\right) = 0$. Concerning $A_{-i}^c(\tau)$, we observe that (i) $\partial_{A_{-i}}\varphi_c^i(\tau, A_{-i}) = \kappa_i''\left(p^{-1}(\tau) - A_{-i}\right) > 0$, (ii) $\varphi_c(\tau, p^{-1}(\tau)) = \tau > 0$ and (iii) $\varphi_c^i(\tau, 0) = \tau - \kappa_i'\left(p^{-1}(\tau)\right) = \phi_c^i(\tau)$ this last function being the same as in Eq. (A.4) but now indexed by agent i. So by using Step 2 of the proof of Lemma 1, we know that $\left(\phi_c^i\right)' > 0$ and that there exists a unique τ_c^i such that $\phi_c^i(\tau_c^i) = 0$. We can therefore say that:

$$\begin{cases} \forall\tau \leq \tau_c^i, \ \exists A_{-i}^c(\tau) \in \left[0, p^{-1}(\tau)\right], \varphi_c\left(\tau, A_{-i}^c(\tau)\right) = 0 \\ \forall\tau > \tau_c^i, \ \varphi_c(\tau, A_{-i}) > 0 \text{ for all } A_{-i} \in \left[0, p^{-1}(\tau)\right) \end{cases}$$

Concerning $A_{-i}^m(\tau)$, we now observe that (i) $\partial_{A_{-i}}\varphi_m^i(\tau, A_{-i}) = \kappa''\left(p^{-1}(\tau) - A_{-i}\right) > 0$, (ii) $\varphi_m^i(\tau, p^{-1}(\tau)) = \tau > 0$ and (iii)$\varphi_m^i(\tau, 0) = \phi_m^i(\tau)$ this last function again being the same as in Eq. (A.4)). Again using Step 2 of the proof of Lemma 1, we have:

$$\begin{cases} \forall\tau \leq \tau_m^i, \ \exists A_{-i}^m(\tau) \in \left[0, p^{-1}(\tau)\right], \varphi_m^i\left(\tau, A_{-i}^m(\tau)\right) = 0 \\ \forall\tau > \tau_m^i, \varphi_m^i(\tau, A_{-i}) > 0 \text{ for all } A_i \in \left[0, p^{-1}(\tau)\right) \end{cases}$$

Finally since $\varphi_m^i(\tau, A_{-i}) < \varphi_c^i(\tau, A_{-i})$ and both are increasing we can say that for $\tau \leq \tau_c^i$, $A_{-i}^c(\tau) < A_{-i}^m(\tau)$.

Step 3: *The unconstrained best response*

Let us concentrate on the last equation of Eq. (C.1). If we compute the associated best response (without regard to $a_i > p^{-1}(\tau) - A_{-i}$) we obtain a standard best response $br_i(A_{-i})$ which corresponds to a Cournot game in which the inverse demand is $p(A)$. This function exists for all $A_{-i} \in \left[0, \varepsilon^{-1}(Q_{\max})\right]$, since (i) $\psi_i(a_i, A_{-i})$ is decreasing in a_i (see Step 1 and recall that $\kappa" > 0$), (ii) $\lim_{a_i \to 0}\psi_i(a_i, A_{-i}) = p(A_{-i})\left(1 + e_p(A_{-i})\right) > 0$ since $e_p > -1$ by assumption and (iii) $\lim_{a_i \to (\varepsilon^{-1}(Q_{\max}) - A_{-i})}\psi_i(a_i, A_{-i}) = -\kappa_i'\left(\varepsilon^{-1}(Q_{\max}) - A_{-i}\right) < 0$ for $A_{-i} < \varepsilon^{-1}(Q_{\max})$ while for $A_{-i} = \varepsilon^{-1}(Q_{\max})$ the best response is $a_i = 0$.

Step 4: *The best response*

Three cases must be distinguished.

Case 1: $\tau > \tau_m^i$

In this case we have $\varphi_c^i(\tau, A_{-i}) > \varphi_m^i(\tau, A_{-i}) > 0$. If we now keep in mind that $(\tau - \kappa'(a_i))$ is decreasing and converges to $\varphi_c^i(\tau, A_{-i})$ as $a_i \to \left(p^{-1}(\tau) - A_{-i}\right)$, $\partial_{a_i}\pi$ only admits a zero in the third case of Eq. (C.1). In other words the best response is $BR_i(A_{-i},\tau) = br_i(A_{-i})$ defined in Step 3.
Case 2: $\tau \in \left(\tau_c^i; \tau_m^i\right]$

Here we know that $\varphi_c^i(\tau, A_{-i}) > 0$ and therefore $\tau - \kappa_i'(a_i) > 0$ for all $a_i < p^{-1}(\tau) - A_{-i}$, but $\exists A_{-i}^m(\tau) \in \left[0, p^{-1}(\tau)\right]$, $\varphi_m^i\left(\tau, A_{-i}^m(\tau)\right) = 0$. This means that the best response is given by:

$$BR_i(A_{-i},\tau) = \begin{cases} p^{-1}(\tau) - A_{-i} \text{ for all } A_{-i} \le A_{-i}^m(\tau) \\ br_i(A_{-i}) \text{ else} \end{cases}$$

Case 3: $\tau \le \tau_c^i$

In this case both thresholds matter so that the best response is given by:

$$BR_i(A_{-i},\tau) = \begin{cases} (\kappa_i')^{-1}(\tau) \text{ for all } A_{-i} < A_{-i}^c(\tau) \\ p^{-1}(\tau) - A_{-i} \text{ for all } A_{-i} \in \left[A_{-i}^c(\tau), A_{-i}^m(\tau)\right] \\ br_i(A_i) \text{ else} \end{cases}$$

Appendix D. Proof of Lemma 4

Existence:

Let us first show that for $\tau < \tau_c^{\min}$, playing $\left(\kappa_i'(p^{-1}(\tau))\right)_{i=1}^n$ is a Nash equilibrium, by showing that it is each player's best response. This follows directly from the definition of $\tau_c^{\min}$. In this case we have $\forall i$, $\tau < \kappa_i'(p^{-1}(\tau))$ or $\forall i$, $(\kappa_i')^{-1}(\tau) < p^{-1}(\tau)$. We can therefore say that:

$$\forall i \quad A_{-i}^c = \sum_{j=1, j\neq i}^{n} \left(\kappa_j'\right)^{-1}(\tau) < p^{-1}(\tau) - (\kappa_i')^{-1}(\tau) = A_{-i}^c(\tau) \tag{D.1}$$

This means from Eq. (25) that playing $a_i^C = (\kappa_i')^{-1}(\tau)$ is a best response for each firm.
Uniqueness:

Concerning uniqueness, let us first observe that the best response (see Eq. (25)) is bounded from above by $(\kappa_i')^{-1}(\tau)$. So if there exists another equilibrium, say b^C, there must be at least one firm i_0 such that $b_{i_0}^C < \left(\kappa_{i_0}'\right)^{-1}(\tau)$ and, due to the upper bound, $B_{-i_0}^c \le \sum_{j=1, j\neq i_0}^n \left(\kappa_j'\right)^{-1}(\tau)$. But this leads to a contradiction, since for $\tau < \tau_c^{\min}$, we have as before that $B_{-i_0}^c < A_{-i}^c(\tau)$, so that $b_{i_0}^C = \left(\kappa_{i_0}'\right)^{-1}(\tau)$ should be the best response.