Trade Liberalization and Heterogeneous Technology Investments.*

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November 2014

Abstract: This paper proposes an intra-industry trade model with heterogeneous firms that incorporates a productivity-enhancing endogenous sunk investment. Closed-form solutions for the general-equilibrium open economy show that a symmetric trade liberalization has two opposite effects on firm-level investment and productivity. Freer trade, by raising export profits, increases the incentives to invest in technology. It also dampens them, however, as profits stemming from domestic sales are reduced. Only exporters benefit from the former positive effect. The shape of the distribution of initial efficiency draws, the level of trade costs as well as the technology intensity of the industry are key elements removing the ambiguities regarding the net impact of trade liberalization on firm- and industry-level productivity. We discuss recent empirical evidence supporting the main mechanisms of the model.

JEL codes: F10, F12 and F15

Keywords: firm-heterogeneity; technology investment; firm productivity; trade liberalization.

*This paper is an updated version of our PSE working paper N° 2006 - 36, entitled "Trade Liberalization and Heterogeneous Within-Firm Productivity Improvements". We are grateful to Gregory Corcos, Swati Dhingra, Lionel Fontagné, Hugo Hopenhayn, Marc Melitz, Philippe Martin, Thierry Mayer and James R. Tybout for helpful discussions at different stages of this work. Of course, we are the sole responsible of the views expressed here as well as of any error.

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1 Introduction

Empirical evidence on the effects of trade liberalization suggests that falling trade costs impact aggregate productivity through several channels. The most commonly emphasized are the reallocation of production activity between firms, firm selection and within-firm productivity growth. Several "new new" trade theory models featuring productivity-heterogeneous producers have been proposed to study the underlying micro level adjustments.[1] Although insightful in analyzing selection and reallocation channels, this theoretical literature has paid much less attention to within-firm mechanisms. In these models, firm-level productivity differences are generally exogenously given and assumed to remain fixed over time. When the possibility of technology adoption has been introduced, it has mostly been modeled as a binary technology choice, where all firms that upgrade their technology enhance their efficiency in the same proportion. Trade liberalization would then homogeneously affect the level of the investment engaged and the productivity growth attained by firms. Empirical studies, however, document unequal effects of trade on firm performances.[2]

This paper attempts to fill this gap by theoretically investigating the relationship between trade liberalization and the "within" component of productivity growth. We analytically study how changes in variable trade costs lead to heterogeneous variations in technology investments that, in turn, determine changes in firm productivity. Our contribution to the existent literature is to bring back to mind a fundamental "Schumpeterian" mechanism: future profits are, unavoidable, a key source of incentives to undertake productivity-enhancing investments. We show that a fall in trade costs affects firm profits in contradicting ways, the net effect of which shapes the incentives to improve productivity depending on firm characteristics, namely export status.

We propose a natural extension of the Melitz's (2003) trade model to formally examine our argument in the simplest way.[3] We add to his setup an intermediate stage of investment in technology that ultimately determines the level of productivity at which firms shall produce in their market life. In our setting, the investment decision is taken over a continuous support, so that the level of investment is an endogenous sunk cost (cf. Sutton, 1991), which is heterogeneous across firms.[4] As in Melitz (2003), after paying a fixed sunk entry cost, new entrants draw an initial level of efficiency from a common distribution. We interpret this initial draw as "potential efficiency". In order to attain any level of actual productivity at which operate in a later stage of production, a firm needs to invest first in a technology input. We assume that the way in which this technology input affects firm productivity positively depends on the potential efficiency of the firm. As the investment decision occurs with a perfect foresee of export activity the model naturally reproduces discontinuities in investment related to export status.

[1] These "new new" trade theory models are summarized by Redding (2011) and Melitz and Redding (2012). Bernard et al (2012) provide a recent survey on related industry- and firm-level evidence. Mayer and Ottaviano (2007) completes the picture with firm-level data on European countries.

[2] See for instance Pavcnik (2002), Trefler (2004), Tybout (2003) and other references discussed in Section 5.

[3] For the sake of tractability we keep here the Melitz's (2003) framework. Melitz (2003) adapts Hopenhayn's (1992) analysis of firm heterogeneity in order to embed it into a Krugman's (1980) model of intra-industry trade. An extension to include pro-competitive effetcs of trade (not present in Krugman, 1980) is proposed by Melitz and Ottaviano (2008). For an alternative analysis of firm heterogeneity in a Ricardian model of trade see Bernard et al. (2003). Bernard, Redding and Schott (2007) also explore firm heterogeneity in an integrated model of monopolistic competition and comparative advantage, of the kind proposed by Helpman and Krugman (1985).

[4] As we note in the next section, this technological investment is different from the quality-upgrading investment featured in recent models of heterogeneity in quality. In our model, we focus on investments related to technical factors that are not exogenously driven by demand-side factors such as preferences for high quality goods.

This framework allows for a complete tractable mapping from the initial draw of potential efficiency to the ex-post endogenous productivity. In other words, the Melitz's (2003) framework is completely nested in the one proposed here, a feature that enormously facilitate the analysis and interpretations. Based on this extended setup, we fully characterize the steady-state general equilibrium of an open economy that trades with a group of symmetric countries. We then analytically derive the consequences of trade liberalization (understood as a symmetric reduction of variable trade costs) on the structure of the industry as well as on firm- and industry-level productivity.

Results confirm the relevance of the aforementioned Schumpeterian insight. On the one hand, by improving foreign-market access trade liberalization increases export profits, which positively impacts technology investments. On the other hand, it reduces domestic-market profits in equilibrium, which plays a negative role on firm-level investment and productivity. While the net effect of trade liberalization can be positive for exporters' productivity, it is theoretically ambiguous. For non-exporters, however, trade liberalization has an unambiguous negative impact on productivity, as they only experience a sharp decline in their profit.

The mechanisms leading to a fall in domestic profits after trade liberalization are of similar nature to those appearing in Melitz (2003) and are ultimately reflected in the intensity of the process of firm selection into the domestic market. Our model grasps new insights on this by showing the interactions between the different sources of industry-level productivity growth. Everething else equal, The domestic threshold of potential efficiency, and hence domestic-market selection, is all the more important in industries with high technology intensity, where productivity sensibly reacts to the technological input. In those industries, productivity-improving investments are larger and the industry as a whole becomes more competitive after the technology investment stage. Hence, the productivity growth that takes place within firms influences the process of entry and the reallocation of market shares between-firms. By the means of general equilibrium interactions that upwardly shift the real wage, the between effect increases domestic-market selection (and so reduces domestic profits), which in turn is negatively associated to the incentives to undertake the technology investment. In these interactions a pecuniary externality is at play, as it is common under monopolistic competition. This implies that, when deciding their level of investment, the monopolistically competitive firms take industry aggregates as given and do not consider the impact of their own investment choice on industry-level productivity, and thus on their own profitability. This influences in turn domestic-market selection.

In such a context, a symmetric trade liberalization rises *exporting firms*' productivity when the negative incentives for investment conveyed in general equilibrium via firm selection are limited. This imposes an upper bound to the elasticity of the domestic threshold of potential-efficiency with respect to variable trade costs. On the other hand, it rises the *aggregate* productivity of the industry when the exit of the lower tale of the potential efficiency distribution is able to generate a sizable increase in the average potential efficiency. This imposes a lower bound to the elasticity of the average potential efficiency with respect to variable trade costs. These conditions can be simultaneously fulfilled under distributions of initial efficiency in which the concentration is large in the lower tale and becomes substantially lower as one moves to the right. In such a case, the required general-equilibrium adjustment of the domstic threshold of potential efficiency would be small but able to induce a considerable trimming of firms.

We illustrate this in an example with Pareto distributed initial draws. This example also helps to discuss how the level of trade cost and the technology intensity of the industry are key in removing the ambiguities regarding the net impact of trade liberalization on exporters' productivity. An interesting insight emerging from this analysis is that micro adjustments can be completely offset at the macro level by composition effects. It might be the case that, even if all firms become less productive after the reduction of trade costs, aggregate productivity rises due to high selection and the reallocation of market shares.

The rest of the paper is organized as follows. The next section briefly presents how our model contributes to the related theoretical literature. For the sake of clarity, we start the formal analysis by characterizing the model in a closed economy in section 3. This allows us to present the main implications of our technology investment stage in the context of the Melitz (2003) framework. In section 4, we extend the model to consider an open economy and present the main results linking the reduction of variable trade costs and productivity gains at both the micro- and the macro-level. We discuss in section 5 how the paper relates to available empirical findings. The last section concludes. All details on formal derivations are relegated to the Technical Appendix accompanying the paper.

2 Related theoretical literature

One may, at least, distinguish three types of models, closely related to the one proposed here.[5] A first class of models considers the above mentioned binary choice technology adoption setting. Firms choose among a high- and a low-productivity technology, with a trade-off between fixed and marginal costs. Such a trade-off is generally supposed to be more advantageous when productivity is higher. Freer trade, by rising expected export profits, induces incentives to adopt the highest technology but only for the most productive firms (e.g. Yeaple, 2005; Bustos, 2011; Navas Ruiz and Sala, 2007; Bas, 2012; Bas and Berthou, 2012). In a nutshell, these binary-choice models focus on the decision to engage in technology adoption, what can be called the extensive margin of investment, while the analysis remains silent regarding the intensive margin, i.e. the level of investment, as all technology-upgrading firms are constrained to undertake the same outlay. By incorporating the latter into the analysis, through an endogenous sunk technology cost, our model is able to capture a negative effect of trade liberalization on technology outlays, since we allow entrants to marginally adjust their level of investment according to variations in their expected profit. This ingredient indeed suffices to reproduce efficiency-reducing effects of trade liberalization within an otherwise standard framework of monopolistic competition and constant elasticity of substitution (CES). [6]

A special case of binary technology-adoption choice is the model proposed by Lileeva and Trefler (2010) to

[5] We only discuss here closely related works. For instance one may get endogenous firm productivity without a productivity-enhancing investment by looking at the endogenous range of products produced in equilibrium in a multiproduct-firm setting (Mayer et al, 2013; Bernard et al., 2011). By pushing firms to drop least-profitable products trade may generate productivity gains at the firm level through a composition effect, in an analogous fashion to the rise in aggregate productivity generated by the exit of least productive firms in Melitz (2003).

[6] Other ingredients of course may also lead to efficiency-reducing effects of trade liberalization. For instance, assuming a linear-demand system of the type used in Melitz and Ottaviano (2008), Spearot (2013) shows that combining variable demand elasticities, heterogeneous firms and capital acquisitions trade liberalization may also have a negative effect on aggregate productivity. Likewise, Baldwin and Robert-Nicoud (2008), in a Melitz's (2003) model extended to include product-innovation-based growth assume knowledge spillovers in the creation of new variaties. Trade liberalization here raises the expected sunk knowledge-investment for new entrants. We present here a complementary explanation, slightly change from the base-line setting of Melitz (2003).

study Canadian firms' reaction to the Canada-US Free Trade Agreement. By letting productivity improvements be heterogeneous among firms, the authors describe technology-adoption and export participation choices in a simple partial-equilibrium model. For Lileeva and Trefler (2010) if two firms with different initial productivity are indifferent between the choices of exporting and investing, on the one hand, and doing neither, on the other, it is because the low-productivity firm expects higher returns from technology investment than the high-productivity one. This rationale is then used in order to explain why small and less-productive plants do export in their data. In our model we also suppose that productivity gains are heterogeneous but, differently to Lileeva and Trefler (2010), they are increasing in initial productivity (what we call potential efficiency).

A second set of models considers an endogenous investment as we do here, but instead of improving efficiency such an investment yields a higher perceived level of quality (e.g. Kugler and Verhoogen, 2012; Johnson, 2012). These models make an interpretation of Melitz (2003) in terms of quality heterogeneity with the help of asymmetric CES preferences. They provide guidance to empirically analyze quality-adjusted prices differences and correlations between prices and firm size. Regarding these works, the main specificity of our analysis is to focus on productivity, as in the original reading of Melitz (2003). From a dynamic point of view, this is no longer a matter of interpretation. A technology allowing for efficiency variations should not be *a priori* governed by demand factors, so that we believe efficiency- and quality-upgrading investments deserve specific exploration. Moreover, our setting follows different modeling choices such as (i) a general treatment of the distribution of productivity draws - instead of assuming Pareto distribution as it is the practice in models of quality-upgrading investment; (ii) a general equilibrium analysis - whereas partial equilibrium is considered by Johnson (2012) and hence no analysis of domestic market selection; and (iii) it incorporates variable trade costs - not present in Kugler and Verhoogen (2012).

Finally, a third type of theoretical analysis focuses on productivity dynamics. Atkeson and Burstein (2010) analyze a model where heterogeneous firms make in each period risky investments in an R&D good in order to improve their productivity (process innovation). The sunk entry cost is measured in units of this research good, so that entry decisions can be interpreted as product innovation. The authors mainly focus on the impact of trade costs on *aggregate* productivity in general equilibrium. They show that a variation in trade costs does generate firm-level adjustment of incumbent firms in terms of their exit, export and process innovation decisions. However, the impact of these adjustments on aggregate productivity is, at least, offset by adjustments in entry decisions. This result is analytically presented in three scenarios which are important for the authors' purpose but have key differences with the one pursued here in this paper as *endogenous* firm selection (into both the domestic and the export markets) and productivity dynamics are not simultaneously considered in their work. They consider analytically (i) the case where all firm exports, i.e. a case with no fixed export costs, (ii) the case where most productive firm exports but there is no productivity dynamics, i.e. the Melitz (2003)'s case and (iii) the case where firms do engage in process innovation, but where the fixed cost structure is simplified to consider zero fixed costs of production, and a Markov transition matrix featuring zero or infinity fixed export cost, i.e. a case where firm selection *does not* depend on productivity as they never pay the fixed costs for producing and exporting. Interestingly, although different in nature, the latter case also leads to heterogeneous reactions depending on export status: a reduction of trade costs increases the incentives for process innovation for exporters and decrease it for non-exporters, if the export status is sufficiently persistent. In our simpler but arguably tractable setting we shed light on these heterogeneous reactions by highlighting how between- and within-firms channels interact when both endogenous firm selection and endogenous productivity improvements are at work. As in Atkeson and Burstein (2010), our framework can also highlight how micro-level adjustments can be offset at the macro-level.[7]

3 The Closed Economy

3.1 Basic Setup

3.1.1 Consumers

Consumers derive utility from a continuum of horizontally differentiated varieties indexed by $\omega \in \Omega$. Preferences are characterized by a constant elasticity of substitution (CES) utility function of the form $U = \left[\int_{\omega\in\Omega} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}$, where $\sigma > 1$ is the elasticity of substitution between two different varieties and Ω the set of available varieties in the industry. Consumption decisions, given the price $p(\omega)$ offered by each producer (one for each variety) and aggregate expenditure $\int_{\omega\in\Omega} p(\omega) q(\omega) d\omega = R$, lead to the usual CES demand $q(\omega) = Q\left[\frac{p(\omega)}{P}\right]^{-\sigma}$, where $P = \left[\int_{\omega\in\Omega} p(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}$ is the welfare-based index price and $Q \equiv U$ the composite good summarizing aggregate consumption, so that $R = PQ$.[8]

3.1.2 Producers and timing

The model considers a three-stage timing including entry, investment and production decisions. At each time period, an unbounded mass of prospective entrants decide whether to enter the market and become active producers. If they decide so, entrepreneurs must pay a fixed sunk entry cost f_e which allows them to draw a level of potential efficiency $\varphi_0 > 0$ from a common probability distribution $h(\varphi_0)$ with support over $[0, \infty)$. Once this initial draw is known, firms decide on a technology investment $I > 0$ that will partially determine the actual level of productivity at which they operate in the stage of production, according to

$$\varphi = \varphi_0 I^{\beta} \tag{1}$$

with $0 < \beta < 1$. The parameter β captures the extent to which firm productivity increases with technology investments in the industry. We think of technology investment as an endogenous sunk investment in a technology input (R&D or customized ICT goods) produced with labor under constant return to scale (CRS) in a perfectly competitive upstream sector. Let γ be the homogeneous labor productivity in that sector. For simplicity, labor is supposed to be fully mobile within the economy so that the same wage prevails. The unit price of the technology

[7] Other models on this line pay greater attention to transition dynamics, not considered in this paper. See for instance Burstein and Melitz (2012), which uses a similar framework to that of Atkeson and Burstein (2010), or Costantini and Melitz (2008), which studies the timing of a (binary) innovation decision relative to the timing of exporting.

[8] In order to facilitate the exposition, round brackets are reserved for functions' arguments.

input is then $p_I = \frac{1}{\gamma}$.

This investment is made once, after entry and before starting production, so that the resulting endogenous productivity level $\varphi > 0$ is the one at which firms will operate during their production life. The random variable φ_0 can then be seen as a heterogeneous non-transferable initial "technology endowment" that partially determines firm productivity.

In the production stage, firms produce an horizontally differentiated variety and engage in monopolistic competition. The technology available to each firm uses labor as the sole input and is characterized by a cost function featuring a fixed overhead cost $f > 0$, and a constant marginal cost that is decreasing in the level of productivity, φ, attained by the firm. Let $l(\varphi, q)$ be the labor required to produce q units of output for a producer whose investment has delivered an ex-post productivity level of φ. The production technology can then be summarized as $l(\varphi, q) = f + \frac{q}{\varphi}$.

Given the constant-elasticity formulation of eq. (1) and the fact that the production function is linear in φ (once the fixed units of labor necessary to start production have been used), the elasticity of firm productivity to technology investment β pins down the technology intensity of the industry for a given vector of input prices. Hence, in order to give more intuition to out results we may refer to a case of high β (everything else being equal) as a case of high technology intensity.[9]

Firm productivity is key in determining profitability. Not all firms that enter the market will produce thereafter since those having drawn a sufficiently low potential efficiency may not be able to pay the fixed production cost, even if they invest in technology. Those firms will exit the market before the stage of production and, rationally, make no investment. In addition to this source of exit, an active producer may suffer a bad productivity shock with exogenous probability δ, in which case it will be forced to cease production. This event arises independently of firm productivity. This process of entry, investment and production (with eventual exit) is featured continuously at every time period.[10] We shall focus on the steady-state general equilibrium where entry flows to and exit flows from active production offset each other.

Hence, we depart from Melitz (2003), henceforth referred to as the base-line model, essentially by considering an intermediate stage of technology investment. In our model, entrepreneurs must invest a sufficient amount in technology if they want to keep or enhance their potential efficiency φ_0. If the investment outlay, I, is not restricted to be greater than unity, eq. (1) would allow for the possibility that some entrepreneurs end-up with a smaller level of efficiency than their initial draw. Although such an outcome will be fully rational given a poor exogenous technology endowment, it may not reflect a general case. Thus, without any implication on our results, efficency downgrading will be ruled out by parameter assumptions.

Note that heterogeneity in the set of firms that remain in the market can be directly indexed by φ. Thus, in order to facilitate the presentation we shall write firm-level variables in the production stage as in the base-line model and derive optimal technology investment as a function of φ. A complete mapping to potential efficiency φ_0

[9] More precisely, considering the linear interval of the production function, the residual demand and the optimal investment decision (given in eq. (3) in subsection 3.1.3) we can write $\frac{I}{[q/\varphi]} = \beta \frac{w}{\delta P_I}$.

[10] The time index is dropped for notational simplicity.

will then be established through eq. (1).

Profit maximization in the production stage leads to the well-known forms of price, revenue and profit functions, respectively:

$$p(\varphi) = \frac{\sigma}{\sigma - 1}\frac{1}{\varphi}, \quad r(\varphi) = \sigma D \varphi^{\sigma-1}, \quad \pi(\varphi) = \frac{r(\varphi)}{\sigma} - f \tag{2}$$

where labor is the numeraire, so that wage has been normalized to one, and $D \equiv \frac{R}{\sigma}\left[\frac{\sigma-1}{\sigma}P\right]^{\sigma-1}$ is an index of residual demand. As we shall see, this index conveys general-equilibrium demand-driven mechanisms affecting firms' profits and so the incentives to engage in productivity upgrading.

3.1.3 Firm value and investment

Consider a firm that has paid the sunk entry cost, knows its potential efficiency and evaluates the decision of investment. Its present value writes $v(\varphi) = \max\left\{0, \sum_{t=0}^{\infty}[1-\delta]^t \pi(\varphi) - p_I I\right\}$, where φ is partly determined by I (see eq. (1)) and $[1-\delta] < 1$ is the exogenous probability of survival, so that δ acts here as a discount rate.

The elasticity of revenue to investment, $\varepsilon \equiv \frac{dr(\varphi)}{dI}\frac{I}{r(\varphi)} = \beta[\sigma - 1] > 0$, is an important determinant of the optimal investment decision, the first-order condition of which can be conveniently written in terms of φ, the level of productivity "targeted" by the investment:

$$I(\varphi) = \frac{\varepsilon}{\delta p_I}\frac{r(\varphi)}{\sigma} \tag{3}$$

an equation that is meaningful only if the second-order condition $0 < \varepsilon < 1$ pertains. The total amount of the technology outlay, $p_I I(\varphi)$, is equal to the present value of a fraction ε of the per-period stream of variable profits, $\frac{r(\varphi)}{\sigma}$. Since optimal investment can be written as a function of the "target" productivity level φ and, by eq. (1), φ depends on the potential efficiency draw φ_0, the endogenous productivity level can be fully characterized as a function of φ_0 only: $\varphi(\varphi_0) = \varphi_0 I(\varphi(\varphi_0))^{\beta}$. We can then use this latter fonction in eq. (3) in order to solve for $I(\varphi(\varphi_0))$. This allows to write the mapping from potential efficiency to the optimal firm-level investment as:

$$I(\varphi(\varphi_0)) = \left[\frac{\varepsilon}{\delta p_I}\frac{r(\varphi_0)}{\sigma}\right]^{\frac{1}{1-\varepsilon}} \tag{4}$$

which is positively influenced by *potential* profitability $r(\varphi_0)$, the revenue that a firm would obtain were its level of productivity remains exactly equal to its potential efficiency. Substitution of (4) into (1), gives the endogenous productivity level

$$\varphi(\varphi_0) = \varphi_0^{\frac{1}{1-\varepsilon}}\Delta \qquad \textit{with } \Delta \equiv \left[\frac{\varepsilon\gamma}{\delta}D\right]^{\frac{\beta}{1-\varepsilon}} \tag{5}$$

As eq. (5) makes clear, demand conditions summarized by D influence the technology investment decision. Namely, any exogenous reduction of the index price, usually interpreted as an increase in the competitiveness of the industry, will also imply a reduction in profits and so a reduction in technology investments and firm-level productivity.

Note also that profitability differences are exacerbated by the possibility of technology investment. Consider, two firms (1 and 2) having different initial draws, $\varphi_0^1 > \varphi_0^2$, the gap between their ex-post endogenous productivity writes $\frac{\varphi(\varphi_0^1)}{\varphi(\varphi_0^2)} = \left[\frac{\varphi_0^1}{\varphi_0^2}\right]^{\frac{1}{1-\varepsilon}}$, and the gap between their revenues $\frac{r(\varphi(\varphi_0^1))}{r(\varphi(\varphi_0^2))} = \left[\frac{\varphi_0^1}{\varphi_0^2}\right]^{\frac{\sigma-1}{1-\varepsilon}}$. Productivity differences are then more important in technology-intensive industries (i.e. in industries with higher β and so higher ε). With β high, differences in potential efficiency may translate into strong profitability differences, even if substitutability among goods is low.[11]

After considering optimal investment, firm value is then given by

$$v\left(\varphi\left(\varphi_0\right)\right) = \frac{1}{\delta}\left\{[1-\varepsilon]\frac{r\left(\varphi\left(\varphi_0\right)\right)}{\sigma} - f\right\} \tag{6}$$

3.1.4 Aggregation

Let $\eta\left(\varphi_0\right)$ be the equilibrium distribution of potential-efficiency draws, defined over $[0,\infty)$ and let M denote the mass of active firms at each time period. Over the support of potential-efficiency draws, aggregation properties of the base-line model also hold here. The economy can be reduced to a one with homogeneous firm productivity where the representative firm (henceforth the average firm) draws a potential-efficiency equal to the average index $\widetilde{\varphi}_0 \equiv \left[\int_0^\infty \varphi_0^{\frac{\sigma-1}{1-\varepsilon}}\eta\left(\varphi_0\right)d\varphi_0\right]^{\frac{1-\varepsilon}{\sigma-1}}$. Using the CES demand function, price in eq. (2) and the endogenous productivity level in eq. (5), the aggregate price index can be expressed as that of the average firm:

$$P = M^{\frac{1}{1-\sigma}}\frac{\sigma}{\sigma-1}\frac{1}{\Delta\widetilde{\varphi}_0^{\frac{1}{1-\varepsilon}}} = M^{\frac{1}{1-\sigma}}p\left(\varphi\left(\widetilde{\varphi}_0\right)\right) \tag{7}$$

Likewise, aggregate revenue and profit can be written as those of the average firm scaled up by the equilibrium mass of firms, respectively: $R = Mr\left(\varphi\left(\widetilde{\varphi}_0\right)\right) \equiv M\overline{r}$ and $\Pi = M\left[\frac{r(\varphi(\widetilde{\varphi}_0))}{\sigma} - f\right] \equiv M\frac{\overline{\pi}}{\sigma}$.

It is instructive to see that the same aggregates can also be written directly, in the stage of production, from the ex-post productivity levels, indexed by φ. Since the probability density of these latter is $\mu\left(\varphi\right) = \frac{\eta(\varphi_0)}{\varphi'(\varphi_0)}$, it can be easily shown that the after-investment average productivity $\widetilde{\varphi} \equiv \left[\int_0^\infty \varphi^{\sigma-1}\mu\left(\varphi\right)d\varphi\right]^{\frac{1}{\sigma-1}}$ is the endogenous productivity of our average firm:[12]

$$\widetilde{\varphi} = \varphi\left(\widetilde{\varphi}_0\right) \tag{8}$$

In other words, the base-line model is completely nested, at both the micro and macro levels. The index $\widetilde{\varphi}$ is the Melitz's (2003) index of average productivity, in his paper taken directly from an exogenous productivity support. Moreover, in the limit case where $\beta = 0$, from eqs. (5) and (8) it immediately follows that $\widetilde{\varphi} = \widetilde{\varphi}_0$.

We can thus follow the standard procedure to solve the model and use appropriate function compositions

[11] In the base-line model this ratio is given by $\frac{r(\varphi^1)}{r(\varphi^2)} = \left[\frac{\varphi^1}{\varphi^2}\right]^{\sigma-1}$, so that for a given *productivity* gap low substitution weakens the consequent *profitability* gap.

[12] More specifically $\varphi\left(\widetilde{\varphi}_0\right) = \widetilde{\varphi}_0^{\frac{1}{1-\varepsilon}}\Delta = \left[\int_0^\infty \varphi_0^{\frac{\sigma-1}{1-\varepsilon}}\Delta^{\sigma-1}\eta\left(\varphi_0\right)\frac{\varphi'(\varphi_0)}{\varphi'(\varphi_0)}d\varphi_0\right]^{\frac{1}{\sigma-1}} = \left[\int_0^\infty \varphi\left(\varphi_0\right)^{\sigma-1}\mu\left(\varphi\right)d\varphi\right]^{\frac{1}{\sigma-1}}$. Alternatively, note that $P = M^{\frac{1}{1-\sigma}}p\left(\widetilde{\varphi}\right)$, $R = Mr\left(\widetilde{\varphi}\right)$, $\Pi = M\left[\frac{r(\widetilde{\varphi})}{\sigma} - f\right]$ (see Melitz, 2003), so that bijection of $p\left(\cdot\right)$ and $r\left(\cdot\right)$ on $\mathbb{R}_+$ naturally leads to (8)

mapping potential efficiency and endogenous productivity at the firm- and industry-levels. This will help to ease the exposition and interpretations.

3.2 Equilibrium

3.2.1 The zero cut-off profit (ZCP) condition

The cut-off level of potential efficiency for active production is the lowest potential efficiency allowing for profitable entry. It can then be defined from firm value as $\varphi_0^* = \inf\{\varphi_0 : v(\varphi(\varphi_0)) > 0\}$. Let $H(\varphi_0)$ be the cumulative distribution of initial draws. In equilibrium, the observed distribution of potential efficiency is therefore $\eta(\varphi_0) = \frac{h(\varphi_0)}{1-H(\varphi_0^*)}$ if $\varphi_0 \geq \varphi_0^*$ and $\eta(\varphi_0) = 0$ otherwise. This means that the average index of potential efficiency is a function of the cut-off level: $\widetilde{\varphi}_0 = \widetilde{\varphi}_0(\varphi_0^*)$.

After using $v(\varphi(\varphi_0^*)) = 0$ with the help of eq. (6) and the ratio of relative revenues of the average firm and the cut-off one, $\frac{r(\varphi(\widetilde{\varphi}_0))}{r(\varphi(\varphi_0^*))} = \left[\frac{\widetilde{\varphi}_0}{\varphi_0^*}\right]^{\frac{\sigma-1}{1-\varepsilon}}$, the average profit reduces to the so-called zero cut-off profit condition (ZCP):

$$\overline{\pi}(\varphi_0^*) = \frac{f}{1-\varepsilon}\left[k(\varphi_0^*) + \varepsilon\right] \qquad with \quad k(\varphi_0^*) \equiv \left[\frac{\widetilde{\varphi}_0(\varphi_0^*)}{\varphi_0^*}\right]^{\frac{\sigma-1}{1-\varepsilon}} - 1 \tag{9}$$

Clearly, in the particular case where $\beta = 0$ we have $\varepsilon = 0$ and the ZCP condition collapses to that of the base-line model.

3.2.2 The free-entry condition

New firms enter the market as long as they expect a non-negative value, net of the entry cost. Thus the free-entry equilibrium of risk neutral entrepreneurs requires the cancellation of the ex-ante expected net value of entry: $[1 - H(\varphi_0^*)]\, v(\varphi(\widetilde{\varphi}_0)) - f_e = 0$. By the above-presented properties of aggregation, the expected value, conditional on successful entry, is the value of a firm that draws a potential efficiency of $\widetilde{\varphi}_0$. Using optimal investment, eq. (3), and firm value, eq. (6), for the average firm we obtain the free-entry condition:

$$\overline{\pi}(\varphi_0^*) = \frac{1}{1-\varepsilon}\left[\frac{\delta f_e}{1 - H(\varphi_0^*)} + f\varepsilon\right] \tag{10}$$

Here also, the base-line model applies for the particular case of $\beta = 0$.

3.2.3 The equilibrium cut-off level of production

The intersection between the free-entry condition (10) and the ZCP condition (9) gives the equilibrium cut-off level φ_0^*. This intersection leads to:

$$j(\varphi_0^*) \equiv k(\varphi_0^*)\left[1 - H(\varphi_0^*)\right] = \frac{\delta f_e}{f} \tag{11}$$

Lemma 1 *Consider $j(\phi)$ $\forall \phi \in \mathbb{R}_+$. As ϕ goes from zero to infinity, $j(\phi)$ decreases from infinity to zero . Moreover,* $\frac{\partial j(\phi)}{\partial \beta} > 0$.

Proof. See Technical Appendix A. ■

We can now establish a central proposition of the closed-economy model.

Proposition 1 *There exists a unique equilibrium threshold of potential efficiency* φ_0^* *allowing to participate in the production stage. This threshold increases with the elasticity of firm productivity to technology investment,* β.

Proof. This follows directly from Lemma 1 and eq. (11). ■

The equilibrium cut-off level of potential efficiency φ_0^* defines the range of firms that participate in the market. It is then a key variable capturing the intensity of market selection in the model. In the $(\phi, j(\phi))$ space, where ϕ belongs to the set of possible values of φ_0^*, the RHS of (11) is a horizontal line whereas LHS, $j(\phi)$ is a curve decreasing from infinity to zero. The intersection of both of these gives the equilibrium threshold φ_0^*. As stated in Lemma 1, any exogenous increase in β will shift $j(\phi)$ to the right, leading to a higher φ_0^*. The process of selection into the domestic market is then stronger in technology-intensive industries.

The equilibrium level of φ_0^* determines in turn the rest of aggregate variables of the model. From this macro accounting we can write aggregate revenue and the number of varieties as (details are presented in the Technical Appendix B):

$$R = L \tag{12}$$

$$M = \frac{R}{\bar{r}} = \frac{L[1-\varepsilon]}{\sigma f[k(\varphi_0^*)+1]} \tag{13}$$

3.2.4 Index price

In our model, the index price not only depends on the number of available varieties but also on firm-level investment decisions. Investment in turn is a function of demand conditions, namely the index price (see eq.(4)). Under monopolistic competition, firms take aggregate variables as fixed and do not take into account the consequences of their own investment decisions on aggregate productivity and so on the index price. Notwithstanding, this latter pins down their own residual demand. Considering this feedback leads to

$$P = M^{\frac{1-\varepsilon}{1-\sigma}} \frac{\sigma}{\sigma-1} \frac{1}{\widetilde{\varphi}_0 \left[\frac{\varepsilon\gamma}{\delta}\frac{L}{\sigma}\right]^{\beta}} = \frac{\sigma}{\sigma-1}\frac{1}{\varphi_0^*}\frac{1}{\Psi} \tag{14}$$

where $\Psi \equiv \left[\frac{L}{\sigma}\right]^{\frac{1}{\sigma-1}} \left[\frac{1-\varepsilon}{f}\right]^{\frac{1-\varepsilon}{\sigma-1}} \left[\frac{\varepsilon\gamma}{\delta}\right]^{\beta}$. The first equality in (14) results from plugging the definition of Δ (see eq. (5) of firm productivity) along with the macro condition $R = L$ (see eq.(12)) into the equilibrium price eq. (7) and solving for P. The second is obtained after using the number of varieties of eq. (13). Eq. (14) becomes identical to the one of the base-line model once we restrict our attention to the case where $\beta \to 0$.[13] Here, the presence of $\beta, \varepsilon, \gamma$ and δ highlight technology determinants of the index price.

[13] See Appendix D.1 in Melitz (2003).

3.3 Endogenous productivity

In this section we discuss the link between market selection and productivity gains at both the micro- and macro-level. This analysis in a closed economy will be useful to grasp insights later in the open-economy setting. It follows from eq. (5) that the resulting firm productivity depends on demand conditions determining profitability, namely the aggregates of price and revenue. Aggregate revenue is fixed at the macro level by the labor endowment, so that the key variable conveying industry-level consequences of firm heterogeneity is the aggregate price. By eq. (14) we know that an increase in the potential efficiency threshold allowing to participate in the market (i.e. a stronger market selection) will lead to a lower index price and so to a higher level of welfare, as in the base-line model. However, as previously mentioned, such a reduction of the index price will also lower expected profits and so the incentives to invest in technology. This latter effect can be interpreted as a traditional Schumpeterian mechanism that implies a negative relationship between the cut-off level of potential efficiency and the ex-post firm productivity.

Substituting the equilibrium index price in eq. (14) into the endogenous productivity level described by eq. (5) along with the macro condition $L = R$ yields to

$$\varphi(\varphi_0) = \varphi_0^{\frac{1}{1-\varepsilon}} \left[\frac{f\gamma\varepsilon}{\delta[1-\varepsilon]} \frac{1}{[\varphi_0^*]^{\frac{\sigma-1}{1-\varepsilon}}} \right]^{\beta} \tag{15}$$

which combined to eq. (8) leads to aggregate productivity in equilibrium as $\widetilde{\varphi} = \varphi(\widetilde{\varphi}_0(\varphi_0^*))$.[14]

Definition 1 *A variation in domestic-market selection captured by φ_0^* is unrelated-to-technical-progress (UTP) if it is not driven by changes in the vector $\mathbf{a} = \{\beta, \sigma, f, \delta\}$, i.e. the exogenous parameters determining firm-level productivity in equilibrium. Otherwise it will be referred to as related-to-technical-progress (RTP).*

Lemma 2 *An UTP increase in market selection unambiguously reduces firm-level productivity. However, it can still lead to higher aggregate productivity if $\frac{\widetilde{\varphi}_0'(\varphi_0^*)}{\widetilde{\varphi}_0(\varphi_0^*)}\varphi_0^* > \varepsilon$.*

Proof. See Technical Appendix C. ■

If an exogenous shock raises firm selection and does not affect other determinants of investment and productivity, it will certainly be associated to lower firm productivity. Otherwise, ambiguities arise. Definition 1 seeks then to precise the scope of a *ceteris-paribus* variation of φ_0^* in our analysis of productivity. An UTP increase in φ_0^* can be driven for instance by an exogenous reduction of the sunk entry cost f_e. Conversely, any change in the parameter vector $\mathbf{a}$ will lead to a RTP variation of φ_0^*.[15] In the base-line model, any increase in the intensity of domestic market selection is by construction UTP (since productivity is completely exogenous) and it always leads to an increase in average productivity. In our setting, an UTP rise in domestic-market selection (in terms of potential efficiency)

[14] Note that no scale effects appear in technology investment, as expected within a monopolistic competition setting with CES preferences (since no scale effects appear in residual demands in equilibrium), although a larger market does increase welfare $\frac{1}{P}$.

[15] The price of the technology input is not present in the equilibrium equation (11) determining φ_0^*. The vector $\mathbf{a}$ then does not include γ, although it is part of productivity determinants.

has an ambiguous impact on aggregate productivity. Its negative consequences on investment through demand-driven profitability can only be counterbalanced in the aggregate if such a rise in selection has a sufficiently large impact on aggregate potential efficiency, which is the (endogenous) technology endowment of the average firm. In other words, φ_0^* participates in two opposite ways in the determination of average firm productivity: (i) negatively through demand-driven profitability incentives, expressed in eq. (15); and (ii) positively, by rising the endowment of potential efficiency of the average firm, $\widetilde{\varphi}_0$.

Note that Definition 1 is firm-level. One of the main insights of the open-economy setting is that a symmetric reduction of variable trade costs will induce an UTP increase in domestic-market selection for firms that only serve the domestic market. For exporters, however, the better access to foreign markets will also positively affect their own technical progress and the negative impact of the rise in φ_0^* can be reversed.

Eq. (15) also allows to define the threshold level, φ_{0I}^*, that separates the firms that enhance their potential efficiency from those initially disadvantaged who, conversely, are not able to maintain their initial draw and choose to become less efficient: $\varphi_{0I}^* = \inf\left\{\varphi_0 : \varphi\left(\varphi_0\right) > \varphi_0\right\}$. This threshold will then measure the extensive margin of technology upgrading and is given by $\varphi_{0I}^* = \varphi_0^* \left[\frac{f\gamma\varepsilon}{\delta[1-\varepsilon]}\right]^{-\frac{1-\varepsilon}{\sigma-1}}$. We focus our presentation on the case where no active firm decides to downgrade its efficiency, i.e. $\varphi_{0I}^* < \varphi_0^*$. Clearly, this is equivalent to set a low enough per-period unit price of the technology input: $\frac{\delta}{\gamma} < \frac{f\varepsilon}{[1-\varepsilon]}$.

4 The open economy

Consider now the model for an open economy that trades with n symmetric countries.[16] Given horizontal differentiation, monopolistic competition and scale economies, trade is intra-industry. For the sake of simplicity the technology sector is considered as an upstream non-traded sector. Utility, $U = \left[\int_{\omega\in\Omega^D} q\left(\omega\right)^{\frac{\sigma-1}{\sigma}} d\omega + \int_{\omega\in\Omega^x} q\left(\omega\right)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}$ is derived from the sets of varieties produced by national firms, Ω^D, and by foreign exporters, Ω^x. Because of symmetry, technological conditions of domestic exporters are similar to those of foreign ones, so that imported varieties are in average sold at the same price (in the point of consumption) than exported varieties.

Trade is allowed but is not free of cost. Selling abroad implies both fixed and variable trade costs. The standard iceberg cost treatment is used for the latter: in order to sell 1 unit of good in a foreign market, $\tau > 1$ units must be shipped. Given the monopolistic price rule, the price in foreign markets $p_x\left(\varphi\right)$ scales up the domestic price $p_d\left(\varphi\right) = \frac{\sigma}{\sigma-1}\frac{1}{\varphi}$ by a factor of τ, so that $p_x\left(\varphi\right) = \tau p_d\left(\varphi\right)$. Export sales revenue $r_x\left(\varphi\right)$ then scales down the domestic one: $r_x\left(\varphi\right) = \tau^{1-\sigma} r_d\left(\varphi\right) = \tau^{1-\sigma}\sigma D\varphi^{\sigma-1}$.

Exporting also requires a fixed investment that is not related to the volume of export sales but rather to the fact of selling in a foreign market (product adaptation, advertising, shipping rules and the like). In order to simplify the presentation, without loss of generality, this fixed investment will be treated as a per-period fixed export cost, denoted f_x.

Firm heterogeneity as well as fixed export costs naturally lead to the selection of most productive firms into

[16] Here also we follow Melitz (2003). For an extension featuring asymmetric technologies between countries, see Demidova (2008).

export activities. In the open economy, the model identifies a cut-off productivity level for profitable export activity, denoted by φ_{0x}^{*}. Let χ be a firm-level binary variable of export status, with $\chi = 1$ identifying the case where the firm exports and $\chi = 0$ the case where it only sells to the domestic market. The total revenue perceived by a firm with productivity φ is then

$$r(\varphi, \chi) = r_d(\varphi)\left[1 + \chi n \tau^{1-\sigma}\right] \tag{16}$$

Given the separability of revenues earned from different markets, the profit of an exporter is the sum of the domestic profit $\pi_d(\varphi) = \frac{r_d(\varphi)}{\sigma} - f$ and the export profit $\pi_x(\varphi) = \frac{r_x(\varphi)}{\sigma} - f_x$ earned from each of the n partner countries.

4.1 Investment in the open economy

The investment stage in the open economy takes into account the expected export profits. The present value after entry of a firm able to obtain an ex-post productivity level φ is now $v(\varphi, \chi) = \max\left\{0, \frac{1}{\delta}\left[\frac{r(\varphi,\chi)}{\sigma} - f - \chi f_x\right] - p_I I\right\}$. By the same arguments put forward in the case of the closed economy, the first-order condition of investment writes $I(\varphi, \chi) = \frac{\varepsilon\gamma}{\delta}\frac{r(\varphi,\chi)}{\sigma}$. Hence, the expectation of export activity discontinuously increases the incentives to invest. Productivity, once optimal decisions are taken into account, is therefore a function of both the initial draw of potential efficiency and the export status $\varphi = \varphi(\varphi_0, \chi)$. Total revenue function has the form $r = r(\varphi(\varphi_0, \chi), \chi)$, which means that relative to non exporters, exporters are more profitable because (i) they sell abroad and (ii) they invest more in productivity-enhancing inputs, so that they sell at lower prices in all markets (domestic and foreign) with consequent higher demands. This is why χ appears twice in the arguments of $r(\cdot)$. The corresponding mapping from the initial draw to the optimal investment writes then

$$I(\varphi(\varphi_0, \chi), \chi) = \left[\frac{\varepsilon\gamma}{\delta}\frac{r(\varphi_0, \chi)}{\sigma}\right]^{\frac{1}{1-\varepsilon}} \tag{17}$$

which allows to write the endogenous productivity level as:

$$\varphi(\varphi_0, \chi) = \varphi_0^{\frac{1}{1-\varepsilon}}\Delta(\chi) \quad \text{ with } \Delta(\chi) \equiv \left[\frac{\varepsilon\gamma D}{\delta}\left[1 + \chi n \tau^{1-\sigma}\right]\right]^{\frac{\beta}{1-\varepsilon}} \tag{18}$$

Hence, in the open economy, after entry, the value of a firm having drawn a potential efficiency of φ_0 is

$$v(\varphi(\varphi_0, \chi), \chi) = \frac{1}{\delta}\left\{[1-\varepsilon]\frac{r(\varphi(\varphi_0, \chi), \chi)}{\sigma} - f - \chi n f_x\right\} \tag{19}$$

4.2 Aggregation in the open economy

The index price of all varieties sold in the market, P, is composed of two sub-indexes: one for domestic varieties, P_D, and the other for imported varieties, P_m. The discontinuity in the level of optimal investment implies that the national aggregate, P_D, is in turn composed of the index price of varieties sold only in the domestic market (produced by domestic non-exporters), P_{nx}, and the index price of varieties sold by domestic exporters, P_x. We need to differentiate them because of the discontinuity of the endogenous productivity level, which is tied to export status. All these indexes are measured in the point of production so that they are based on the firms' price charged in their own domestic market $p_d(\cdot)$. The index P_m must then be scaled up by the number of foreign countries and trade costs. Under symmetry between countries, $P_m = P_x$. We have then

$$\begin{aligned} P &= \left[P_D^{1-\sigma} + n\tau^{1-\sigma} P_x^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = M_t^{\frac{1}{1-\sigma}} p_d\left(\varphi\left(\widetilde{\varphi}_0^t, 0\right)\right) \\ & \quad \textit{with} \\ P_D &= \left[P_{nx}^{1-\sigma} + P_x^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = M^{\frac{1}{1-\sigma}} p_d\left(\varphi\left(\widetilde{\varphi}_0^D, 0\right)\right) \\ P_{nx} &= M_{nx}^{\frac{1}{1-\sigma}} p_d\left(\varphi\left(\widetilde{\varphi}_0^{nx}, 0\right)\right) \\ P_x &= M_x^{\frac{1}{1-\sigma}} p_d\left(\varphi\left(\widetilde{\varphi}_0^{x}, 1\right)\right) \end{aligned} \tag{20}$$

Consistent definitions of average potential efficiency have been applied in (20). They are representative of different scopes of aggregation: for the full set of varieties available in the industry $\widetilde{\varphi}_0^t = \widetilde{\varphi}_0^t\left(\varphi_0^*, \varphi_{0x}^*, \tau\right)$; for the subset of domestic varieties sold in the industry $\widetilde{\varphi}_0^D = \widetilde{\varphi}_0^D\left(\varphi_0^*, \varphi_{0x}^*, \tau\right)$; for the subset of domestic varieties produced by non-exporters, $\widetilde{\varphi}_0^{nx} = \widetilde{\varphi}_0^{nx}\left(\varphi_0^*, \varphi_{0x}^*\right)$; and for the subset of domestic varieties produced by exporters $\widetilde{\varphi}_0^{x} = \widetilde{\varphi}_0^{x}\left(\varphi_{0x}^*\right)$. The form of these aggregates and details of calculations are given in the Technical Appendix D. The mass of all varieties available in the industry has been denoted by M_t, which is equal to the sum of domestic varieties, M, and imported ones M_m. The former is the sum of the mass of non-exporters M_{nx} and the mass of exporters, M_x. By symmetry $M_m = nM_x$ so that $M_t = M + nM_x$. As variable trade costs participate in the determination of productivity, they are explicitly considered in the arguments of the aggregates $\widetilde{\varphi}_0^t$ and $\widetilde{\varphi}_0^D$.

Aggregation of (16) allows to express the average revenue and the average profit earned by domestic firms as:

$$\overline{r} = r_d\left(\varphi\left(\widetilde{\varphi}_0^D, 0\right)\right) + M_x n r_x\left(\varphi\left(\widetilde{\varphi}_0^x, 1\right)\right), \quad \overline{\pi} = \pi_d\left(\varphi\left(\widetilde{\varphi}_0^D, 0\right)\right) + \lambda_x n \pi_x\left(\varphi\left(\widetilde{\varphi}_0^x, 1\right)\right) \tag{21}$$

where $\lambda_x \equiv \frac{1-H(\varphi_{0x}^*)}{1-H(\varphi_0^*)} = \frac{M_x}{M}$ is the probability of exporting activity conditional on successful entry, $\pi_d\left(\varphi\left(\widetilde{\varphi}_0^D, 0\right)\right)$ and $r_d\left(\varphi\left(\widetilde{\varphi}_0^D, 0\right)\right)$ the average profit and revenue earned from the domestic market, and $\pi_x\left(\varphi\left(\widetilde{\varphi}_0^x, 1\right)\right)$ and $r_x\left(\varphi\left(\widetilde{\varphi}_0^x, 1\right)\right)$ the average profit and revenue earned from each foreign market. Finally, we can also summarize the average revenue earned by all domestic firms by a single average index of potential efficiency $\widetilde{\varphi}_0^r \equiv \left[\frac{M_t}{M}\right]^{\frac{1-\varepsilon}{\sigma-1}} \widetilde{\varphi}_0^t$, such that $\overline{r} = \frac{R}{M} = r_d\left(\varphi\left(\widetilde{\varphi}_0^r, 0\right)\right)$.

4.3 Equilibrium in the open economy

4.3.1 The ZCP condition in the open economy

In order to state the ZCP condition in the open economy, we adapt the standard procedure to our setting. As usually, we start by tying the average profit of each type of sales (from the domestic and export markets) to their respective cut-off level of potential efficiency (φ_0^* and φ_{0x}^*). We derive then a simple relationship linking φ_{0x}^* to φ_0^* so that, by eq. (21), the ZCP condition in the open economy will define a relationship between the average profit $\overline{\pi}$ and the cut-off of potential efficiency for profitable production, φ_0^*, only. The specific features of our endogenous-productivity assumption will show up in aggregate indexes as well as in the definition of the threshold of potential efficiency for export activity.

As in the closed economy, we exploit the ratio of revenues of two firms with different potential efficiency. For the case of the average profit earned from domestic sales (of all firms, exporters and non-exporters), we consider the domestic revenue of the average firm $\widetilde{\varphi}_0^D$ relative to that of the cut-off firm having drawn φ_0^*. A similar routine applies for the average export profit, where $\widetilde{\varphi}_0^x$ and φ_{0x}^* are put into relationship. We have then:[17]

$$\pi_d\left(\varphi\left(\widetilde{\varphi}_0^D,0\right)\right)=\left[\frac{\widetilde{\varphi}_0^D}{\varphi_0^*}\right]^{\frac{\sigma-1}{1-\varepsilon}}\frac{r_d\left(\varphi\left(\varphi_0^*,0\right)\right)}{\sigma}-f,\quad \pi_x\left(\varphi\left(\widetilde{\varphi}_0^x,1\right)\right)=\left[\frac{\widetilde{\varphi}_0^x}{\varphi_{0x}^*}\right]^{\frac{\sigma-1}{1-\varepsilon}}\frac{r_x\left(\varphi\left(\varphi_{0x}^*,1\right)\right)}{\sigma}-f_x \tag{22}$$

The domestic revenue of the least-productive producer is fixed by its zero after-entry value, which means that its operating (or variable) profit, net of technology investment, exactly pays the fixed overhead cost of production:

$$v\left(\varphi\left(\varphi_0^*\right),0\right)=0\Longleftrightarrow r_d\left(\varphi\left(\varphi_0^*,0\right)\right)=\frac{\sigma f}{[1-\varepsilon]} \tag{23}$$

Likewise, the export revenue of the least-productive exporter, having drawn φ_{0x}^*, is fixed at the margin by its indifference between being an exporter or just a domestic seller. For this firm, the surplus of operating profit (here also, net of technology investment) earned from export activity allows to exactly pay the fixed per-period cost of exporting. Using (19), (16) and (18), this indifference condition can be written in order to fix the per-destination export revenue of the φ_{0x}^* marginal firm[18]:

$$v\left(\varphi\left(\varphi_{0x}^*,1\right),1\right)=v\left(\varphi\left(\varphi_{0x}^*,0\right),0\right)\Leftrightarrow r_x\left(\varphi\left(\varphi_{0x}^*,1\right)\right)=\sigma f_x\left[\frac{1-z}{1-\varepsilon}\right] \tag{24}$$

where the use of $z\equiv\frac{\left[1+n\tau^{1-\sigma}\right]^{\frac{\varepsilon}{1-\varepsilon}}-1}{\left[1+n\tau^{1-\sigma}\right]^{\frac{1}{1-\varepsilon}}-1}<1$ shall prove convenient for the equilibrium analysis.

Different from the base-line model, with an endogenous sunk technology investment the indifference condition of export activity requires consideration of both domestic and export revenues. A firm that anticipates export activity makes a larger technology investment than a non exporter. It becomes then more productive (see eq.(18)) and earns a larger revenue, not only because it enjoys foreign sales but also because it charges a lower price in *all* markets.

[17] We are implicitly focusing our attention on an equilibrium with export sorting, so that the cut-off firm does not export and, thereby, invests an amount of $I\left(\varphi\left(\varphi_0^*,0\right),0\right)$,which allows it to obtain a revenue equal to $r_d\left(\varphi\left(\varphi_0^*,0\right)\right)$.

[18] The routine of calculation is as follows. From eq. (19) the indifference condition of the marginal exporting firm writes $[1-\varepsilon]\left[\frac{r(\varphi(\varphi_{0x}^*,1),1)}{\sigma}-\frac{r(\varphi(\varphi_{0x}^*,0),0)}{\sigma}\right]=nf_x$. Eq. (18) helps to link the endogenous productivity of an exporter with that of a non-exporter: $\varphi\left(\varphi_0,1\right)=\varphi\left(\varphi_0,0\right)\frac{\Delta(1)}{\Delta(0)}$, so that from eq.(16) we can relate the revenue of both types of firms: $r\left(\varphi\left(\varphi_0,1\right),1\right)=r\left(\varphi\left(\varphi_0,0\right),0\right)\left[1+n\tau^{1-\sigma}\right]^{\frac{1}{1-\varepsilon}}$. Finally, note that eq.(16) can be used to write $r\left(\varphi,1\right)=r_x\left(\varphi\right)\frac{\left[1+n\tau^{1-\sigma}\right]}{\tau^{1-\sigma}}$. Combination of these insights leads to eq. (24).

We have now all the pieces to write the ZCP condition in the open economy. Using eq. (22), along with eqs. (23) and (24) into eq. (21), we get

$$\begin{aligned} \overline{\pi} &= \frac{f}{1-\varepsilon}\left[k_d\left(\varphi_0^*, \varphi_{0x}^*, \tau\right)+\varepsilon\right]+n\lambda_x f_x\left[1-z\right]\left[k_x\left(\varphi_{0x}^*\right)+\frac{\varepsilon-z}{1-z}\right] \\ & \quad with \\ k_d\left(\varphi_0^*, \varphi_{0x}^*, \tau\right) &\equiv \left[\frac{\widetilde{\varphi}_0^D\left(\varphi_0^*, \varphi_{0x}^*, \tau\right)}{\varphi_0^*}\right]^{\frac{\sigma-1}{1-\varepsilon}}-1 \\ k_x\left(\varphi_{0x}^*\right) &\equiv \left[\frac{\widetilde{\varphi}_0^x\left(\varphi_{0x}^*\right)}{\varphi_{0x}^*}\right]^{\frac{\sigma-1}{1-\varepsilon}}-1= \ k\left(\varphi_{0x}^*\right) \end{aligned} \tag{25}$$

The last equality highlights that the function $k_x(\cdot)$ is identical to $k(\cdot)$ used in the closed economy model (see eq. (9)), which will facilitate technical analysis.

For the RHS of (25) be a function of φ_0^* we must add a relationship linking both thresholds. This can be achieved by using (23) and (24), with eq. (18) helping to relate the endogenous productivity of exporter and non-exporter firms:

$$\varphi_{0x}^* = \varphi_0^* \rho \quad with\ \rho \equiv \left[\frac{nf_x}{f\left\{\left[1+n\tau^{1-\sigma}\right]^{\frac{1}{1-\varepsilon}}-1\right\}}\right]^{\frac{1-\varepsilon}{\sigma-1}} \tag{26}$$

Therefore, partitioning of export status, such that $\varphi_{0x}^* > \varphi_0^*$, arises whenever:

$$\frac{f_x}{f} > \frac{\left[1+n\tau^{1-\sigma}\right]^{\frac{1}{1-\varepsilon}}-1}{n} \tag{27}$$

Interestingly, differently from the base-line model, here the number of trade partners influences the gap between both thresholds, ρ. It is only when $\varepsilon = 0$ that n does not participate in (26).

4.3.2 The threshold of potential efficiency in the open economy

The free-entry condition in the open economy is analogous to that of the closed economy as it imposes to the average value, regardless the form of the average revenue, to verify, ex-ante, a zero expected net value of entry. Formally, consider the potential-efficiency index $\widetilde{\varphi}_0^r$ that allows to summarize average revenue of the full set of domestic firms. Free entry implies $\left[1-H\left(\varphi_0^*\right)\right] v\left(\varphi_0\left(\widetilde{\varphi}_0^r, 0\right), 0\right) = f_e$, which yields to (eq. (10)). The equilibrium threshold φ_0^* is then given by the intersection of this free-entry condition and the ZCP condition (25), which leads to:

$$\begin{aligned} J\left(\varphi_0^*, \varphi_{0x}^*, \tau\right) &= \delta f_e \qquad (28)\\ &\quad with \\ J\left(\varphi_0^*, \varphi_{0x}^*, \tau\right) &\equiv f j_d\left(\varphi_0^*, \varphi_{0x}^*, \tau\right) + n f_x \left[1-z\right]\left[j_x\left(\varphi_{0x}^*\right) + j_z\left(\varphi_{0x}^*\right)\right] \\ j_d\left(\varphi_0^*, \varphi_{0x}^*, \tau\right) &\equiv \left[1 - H\left(\varphi_0^*\right)\right] k_d\left(\varphi_0^*, \varphi_{0x}^*, \tau\right) \\ j_x(\varphi_{0x}^*) &\equiv \left[1 - H\left(\varphi_{0x}^*\right)\right] k_x\left(\varphi_{0x}^*\right) = j(\varphi_{0x}^*) \\ j_z\left(\varphi_{0x}^*\right) &\equiv \left[\frac{\varepsilon - z}{1-z}\right]\left[1 - H\left(\varphi_{0x}^*\right)\right] \end{aligned}$$

Lemma 3 *Consider the function* $J\left(\phi, \phi_x\left(\phi\right), \tau\right)$ $\forall \phi \in \mathbb{R}_+$*, with* $\phi_x\left(\phi\right) = \rho\phi$*. As* ϕ *goes from zero to infinity,* $J\left(\phi, \phi_x\left(\phi\right), \tau\right)$ *decreases from infinity to zero. Moreover,* $\frac{\partial J}{\partial \beta} > 0$.

Proof. See Technical Appendix E. ■

With this result at hand we can establish the existence and uniqueness of the general equilibrium in the open economy. The ensuing macro equilibrium is presented in the Technical Appendix F. Note that all underlying equations of this open-economy equilibrium also reproduce the base-line model as the limit case when $\beta \to 0$, since $\lim_{\beta \to 0} \varepsilon = \lim_{\varepsilon \to 0} z = 0$.

Proposition 2 *There exists a unique equilibrium threshold of potential efficiency* φ_0^* *allowing to participate in the production stage in the open economy. This threshold increases with the elasticity of firm productivity to technology investment,* β.

Proof. This follows directly by Lemma 3 and eq. (28). ■

As in the closed economy, the macro equilibrium variables are fully determined by φ_0^*. Total revenue is still set by labor size, $R = L$ and the number of available varieties is now

$$M_t = \frac{\left[1 - \varepsilon\right] L}{\sigma f}\left[\frac{\varphi_0^*}{\widetilde{\varphi}_0^t}\right]^{\frac{\sigma-1}{1-\varepsilon}} \qquad (29)$$

4.3.3 The index price, selection and productivity in the open economy

We now turn to writing the endogenous productivity in the open-economy general equilibrium as a function of the φ_0^*. To do so we need first to determine the index price. Here also, in the open economy, there is a micro-macro feedback determining aggregate price: firms' price is partly determined by P through the productivity-enhancing investment decision. Therefore, in order to write P as a function of the equilibrium cut-off φ_0^* we follow a similar routine to that used in the closed economy. This combines aggregate revenue $R = L$ with eqs. (18), (20) and (29) to yield:

$$P = M_t^{\frac{1-\varepsilon}{1-\sigma}}\left[\frac{\sigma}{\sigma-1}\right]\frac{1}{\widetilde{\varphi}_0^t\left[\frac{\varepsilon\gamma}{\delta}\frac{L}{\sigma}\right]^{\beta}} = \left[\frac{\sigma}{\sigma-1}\right]\frac{1}{\varphi_0^*}\frac{1}{\Psi} \qquad (30)$$

where $\Psi \equiv \left[\frac{L}{\sigma}\right]^{\frac{1}{\sigma-1}} \left[\frac{1-\varepsilon}{f}\right]^{\frac{1-\varepsilon}{\sigma-1}} \left[\frac{\varepsilon\gamma}{\delta}\right]^{\beta}$. Not surprisingly, the relationship between the index price and the potential efficiency cut-off has the same form than in the closed economy (see eq. (14)). Hence, the impact of trade liberalization on the index price can be entirely analyzed through the determinants of the threshold level φ_0^* in the open economy.

Using (30), the endogenous firm productivity, eq. (18), can be recast as:

$$\varphi\left(\varphi_0, \chi\right) = \varphi_0^{\frac{1}{1-\varepsilon}} \left[\frac{f\gamma\varepsilon}{\delta\left[1-\varepsilon\right]} \frac{\left[1+\chi n \tau^{1-\sigma}\right]^{\frac{1}{1-\varepsilon}}}{\left[\varphi_0^*\right]^{\frac{\sigma-1}{1-\varepsilon}}}\right]^{\beta} \tag{31}$$

and aggregate productivity as $\widetilde{\varphi}^t = \varphi\left(\widetilde{\varphi}_0^t\left(\varphi_0^*, \varphi_{0x}^*, \tau\right), 0\right)$.

Eq. (31) also allows to identify the cut-off of potential efficiency that allows for effective productivity enhancement, defined now as $\varphi_{0I}^* = \inf\left\{\varphi_0 : \varphi\left(\varphi_0, \chi\right) > \varphi_0\right\} = \frac{\varphi_0^*}{\left[1+\chi n \tau^{1-\sigma}\right]^{\frac{1}{\sigma-1}}} \left[\frac{\delta[1-\varepsilon]}{f\gamma\varepsilon}\right]^{\frac{1-\varepsilon}{\sigma-1}}$. As in the closed economy, an equilibrium where the firm drawing φ_{0I}^* is not able to enter the market requires $\frac{\delta}{\gamma} < \frac{\varepsilon f}{[1-\varepsilon]}$, which we also assume here.

It follows from eq. (31) that the set of parameters that directly determines productivity is different depending on export status. Hence, in the sense of Definition 1, a shift in domestic-market selection induced by a variation in trade costs τ will be UTP for non-exporting firms whereas for exporting firms it will be RTP. As Lemma 1 anticipated, this implies an important asymmetry in the way in which exporting and non-exporting firms react to a reduction of variable trade costs τ. Lemma 1 also announced that the nature of the ensuing productivity adjustments shall differ between the micro and the macro level. The following subsections provide the details of these insights.

4.4 Trade liberalization

The impact of a symmetric trade liberalization, modeled as a reduction of variable trade costs τ, can be analyzed through implicit differentiation of equation (28). Although it should be kept in mind that in equilibrium φ_0^* depends on the full set of parameters participating in (28), we shall focus on τ and consider the implicit relationships $\varphi_0^* = \varphi_0^*(\tau)$ and $\varphi_{0x}^* = \varphi_{0x}^*\left(\varphi_0^*(\tau), \rho(\tau)\right)$ with ρ defined as in eq. (25). The following lemma states a key result characterizing the elasticity of the threshold level φ_0^* with respect to τ.

Lemma 4 *In the open economy, the equilibrium threshold* φ_0^* *satisfies* $0 < -\frac{d\varphi_0^*}{d\tau}\frac{\tau}{\varphi} < \frac{d\rho}{d\tau}\frac{\tau}{\rho} < 1$.

Proof. See Technical Appendix G. ■

We can now summarize the impact of a fall in τ on the different potential-efficiency thresholds.

Proposition 3 *A symmetric reduction of variable trade costs, τ, (i) increases the threshold of potential efficiency for profitable production, φ_0^*; (ii) reduces that for profitable export activity, φ_{0x}^*; and (iii) increases the one for productivity-enhancing investment, φ_{0I}^*, in the same proportion than φ_0^*, when $\varphi_{0I}^* < \varphi_{0x}^*$.*

Proof. From Lemma 4, we have $\frac{d\varphi_0^*}{d\tau} < 0$. Moreover $\frac{d\varphi_{0x}^*}{d\tau} = \frac{\partial\varphi_{0x}^*}{\partial\varphi_0^*}\frac{d\varphi_0^*}{d\tau} + \frac{\partial\varphi_{0x}^*}{\partial\rho}\frac{d\rho}{d\tau} > 0$ is fulfilled whenever $\frac{d\rho}{d\tau}\frac{\tau}{\rho} > -\frac{d\varphi_0^*}{d\tau}\frac{\tau}{\varphi_0^*}$, which has also been established in Lemma 4. Finally, by the same arguments $\frac{\partial\varphi_{0I}^*|_{\chi=0}}{\partial\tau} = \frac{\partial\varphi_{0I}^*}{\partial\varphi_0^*}\frac{d\varphi_0^*}{d\tau} = \frac{d\varphi_0^*}{d\tau} < 0$. ■

A fall in variable trade costs reduces the market share of all domestic sellers, who must operate in a market where a new bundle of imported varieties is available. It also increases the real wage since the labor demand is upwardly shifted by the labor requirements of new profit-seeking entrants to the domestic and to the export markets. With a perfectly inelastic labor supply, the adjustment is entirely made by the real wage, which reinforces firm selection. All these general-equilibrium effects of trade liberalization are inherited from the base-line model. In our setting with technology investment, trade liberalization also impacts firm productivity. On the one hand, a variable trade costs reduction brings down domestic residual demands and so profits, which in turn lessens the incentives to invest in technology. On the other hand, it expands export profits, positively impacting investment. This latter force, however, is only at work for exporting firms as we previously saw in eq. (31). Non-exporters only face an UTP increase in domestic-market selection and therefore a fall in their productivity. Proposition 4 formalizes these results.

Proposition 4 *A reduction of variable trade costs, τ, (i) unambiguously reduces productivity of non-exporters, (ii) leads to productivity improvements of exporters when domestic market selection is highly inelastic with respect to variable trade costs,*

$$-\frac{d\varphi_0^*}{d\tau}\frac{\tau}{\varphi_0^*} < \frac{n\tau^{1-\sigma}}{n\tau^{1-\sigma}+1} \tag{32}$$

and (iii) rises the endogenous aggregate productivity $\widetilde{\varphi}^t$ when the elasticity of the average potential efficiency with respect to variable trade costs is sufficiently high relative to that of domestic market selection.

$$\frac{-\frac{d\widetilde{\varphi}_0^t}{d\tau}\frac{\tau}{\widetilde{\varphi}_0^t}}{-\frac{d\varphi_0^*}{d\tau}\frac{\tau}{\varphi_0^*}} > \varepsilon \tag{33}$$

Proof. See Technical Appendix H. ■

Proposition 4 calls for further attention to the shape of the probability density function governing initial draws of potential efficiency. After a reduction of variable trade costs, productivity gains for exporters arise when the shift of φ_0^* is small enough (cf. condition (32)), so that the negative impact of the increase of this cutoff on investment is limited. At the aggregate level, productivity gains are possible when the positive impact of trade liberalization on the average potential efficiency $\widetilde{\varphi}_0^t$ (through the exit of low potential-efficiency firms and consecutive reallocation of market shares) is large relative to its impact on the threshold level of potential efficiency φ_0^* (cf. condition (33)). Of course, the fulfillment of (32) makes more likely that of (33).

To put it simply, in order to observe productivity gains simultaneously at both the micro (for exporting firms) and macro levels, the upward shift of the threshold φ_0^* induced by trade liberalization should be small but able to push to the exit a sufficiently large proportion low potential-efficiency firms. This can be likely the case when the distribution of potential efficiency is highly concentrated in the lower tail and rather flat and decreasingly concentrated as one moves to the upper tale. When a large proportion of firms draws poor levels of potential efficiency, even a small increase in φ_0^* may lead to a large trimming of firms. In this case, the reduction of domestic profits will be limited, while the increase in $\widetilde{\varphi}_0^t$ can still be sizable. The positive incentives to invest may then predominate in exporters' decisions and the enhanced average potential efficiency may counterbalance the fall in non-exporter productivity.

Why should we expect a small variation of φ_0^* in such a case? To see it, let us rewrite the inequality (32) as:

$$\frac{\partial \Phi^{nx}}{\partial \varphi_{0x}^*} \frac{\varphi_{0x}^*}{\Phi^{nx}} < \frac{1}{\varepsilon} \tag{34}$$

where $\Phi^{nx} \equiv \left[\frac{1}{1-H(\varphi_0^*)} \int_{\varphi_0^*}^{\varphi_{0x}^*} \varphi_0^{\frac{\sigma-1}{1-\varepsilon}} h(\varphi_0)\, d\varphi_0 \right]^{\frac{1-\varepsilon}{\sigma-1}}$ measures the non-exporters' contribution to aggregate potential efficiency (see the proof of Proposition 4 in the Technical Appendix H for details). Consider the case of a marginal reduction of τ that in turns marginally reduces φ_{0x}^*. Condition (34) says that Φ^{nx} should not be substantially reduced after subtracting from the non-exporter set the marginal "slice" of new exporting firms. Intuitively, the contraction of the upper tale of the non-exporter distribution (due to trade liberalization) should not cause a significant reduction in their average "technology endowment", so that they could better resist the impact of trade liberalization. Hence, with $h(\varphi_0)$ featuring a quite small proportion of high potential-efficiency firms and so new exporter candidates, this requirement may indeed be fulfilled.

Two further remarks can be made from (34) regarding the ambiguities of the impact of trade liberalization on exporter's productivity. First, although the net impact can be positve, this possibility is less plausible the lower the level of variable trade costs already attained. Recall that φ_{0x}^* is lower the smaller the level of variable trade costs τ. In distributions increasingly concentrated at the left, when τ is already low, the relative mass of new exporters, after a further reduction of τ, might be large enough to cause an important contraction of Φ^{nx}. Second, as ε goes to zero, the RHS of (34) goes to infinity whereas the LHS remains positive and bounded. This means that exporters' productivity gains induced by trade liberalization are more likely to arise in low- rather than high-technology intensive industries (i.e. industries with high β). This is consistent with the result given proposition 2, whereby more technology intensive industries should experience, *ceteris paribus*, greater selection, a context for strong Schumpeterian effects.

Finally note that even in the case of firm-level productivity loss of *all* firms (exporters and non-exporters), at the industry level things might be compensated by strong selection and reallocation, which rises $\widetilde{\varphi}_0^t$. What is interesting is that such a selection is obtained precisely thanks to the exit of low-productivity firms that would have been more productive if trade liberalization had not taken place.

In the next subsection we rely on the particular assumption of Pareto distribution of the initial potential efficiency in order to illustrate these mechanisms.

4.5 Pareto distribution

Assume that firms draw their initial potential efficiency from a Pareto density function of the form $h(\varphi_0) = \alpha \frac{\varphi_{0\min}^{\alpha}}{\varphi_0^{\alpha+1}}$, where $\varphi_{0\min} > 0$ is the lower bound of the support of potential efficiency and α the shape parameter. The Pareto cumulative distribution function is then $H(\varphi_0) = 1 - \left(\frac{\varphi_{0\min}}{\varphi_0}\right)^{\alpha}$. In order to ease notations let us set $b \equiv \frac{\sigma-1}{1-\varepsilon}$. We assume $\alpha > b$ with the aim of ensuring finite mean of variable profits. Applying this to the equilibrium equation (28) we obtain the equilibrium threshold of potential efficiency for profitable production as[19]

$$\varphi_0^{*\alpha} = \frac{\varphi_{0\min}^{\alpha}}{\delta f_e} \left\{ f \left[\frac{b}{\alpha - b}\right] + n f_x \rho^{-\alpha} \left[\frac{b}{\alpha - b} + \varepsilon\right] \right\} \tag{35}$$

The Pareto assumption allows to illustrate the main properties of this threshold, previously obtained in a more general stochastic setting. Simple inspection reveals that φ_0^* increases with β as it rises the elasticity of revenue to investment $\varepsilon < 1$ and lowers ρ. Moreover, since $\frac{d\rho}{d\tau} > 0$ a reduction of trade variable costs clearly increases φ_0^*. Finally, it also follows immediately from (35) that when $\beta = 0$ (i.e. investment does not affect productivity), the potential efficiency threshold is exactly the productivity threshold of the Melitz model solved under Pareto distribution.[20]

This parametrization also allows to illustrate the key role played by the shape of the probability density function in predicting the impact of a reduction of variable trade costs τ on exporters' productivity. We can restate the sufficient condition (34) for this impact to be positive as

$$\frac{\alpha - b}{b\left[\rho^{\alpha - b} - 1\right]} < \frac{1}{\varepsilon} \tag{36}$$

where the RHS is $\frac{\partial \Phi^{nx}}{\partial \varphi_{0x}^*} \frac{\varphi_{0x}^*}{\Phi^{nx}} = \frac{1}{b} \frac{h(\varphi_{0x}^*)[\varphi_{0x}^*]^{b+1}}{\int_{\varphi_0^*}^{\varphi_{0x}^*} \varphi_0^b h(\varphi_0) d\varphi_0}$ once a Pareto distribution is assumed. As $\rho > 1$, for sufficiently high values of α this condition is likely to be fulfilled. Larger values of α precisely imply that the lower tail of the distribution of potential efficiency is highly concentrated. Moreover, the role of trade costs already noted for the general case is also more visible here. The presence of ρ in (36) which increases with τ, informs us that, after a reduction of trade costs, exporters' productivity gains are more likely to be observed when the economy was rather protected. As the level of trade costs decreases, the rest of parameter being fixed, (36) becomes less likely to be verified. Likewise, everything else being equal, in low technology intensive industries, where ε is low, (36) is likely to pertain.

Figure 1, illustrates this by plotting trade costs τ against the RHS of (36) (the horizontal solid line) and its LHS (the decreasing dashed line). Parameters are set at $n = 5$, $\sigma = 5$, $\beta = \varepsilon / [\sigma - 1]$, $f_x = 1.1f$, $\alpha = b + 0.1$. We have then $\alpha > b$ and by choosing $\varepsilon \in]0,1[$ we have also $\beta \in]0,1[$. Moreover, within this parameter setting and the range of trade costs considered it is always the case that $\rho > 1$. Two cases are illustrated. The first case, in Figure

[19] Intermediate calculations yield $\lambda_x = \frac{1-H(\varphi_{0x}^*)}{1-H(\varphi_0^*)} = \rho^{-\alpha}$, $\widetilde{\varphi}_0^D = \varphi_0^* \left[\frac{\alpha}{\alpha-b}\right]^{\frac{1}{b}} \left[1 + z \frac{n f_x}{f} \rho^{-\alpha}\right]^{\frac{1}{b}}$, $\widetilde{\varphi}_0^x = \varphi_{0x}^* \left[\frac{\alpha}{\alpha-b}\right]^{\frac{1}{b}}$ where it is worth recalling $z = \frac{f}{n f_x} \rho^b \left\{\left[1 + n\tau^{1-\sigma}\right]^{\frac{\varepsilon}{1-\varepsilon}} - 1\right\}$.

[20] See Melitz and Redding (2014), section 6.1. with $\alpha \equiv k$ and the number of trade partners $N - 1$ in their notations. That version of the baseline model does not consider dynamics, so that the "death " shock probability δ does not appear.

1.a, is one of "high" technology intensity and so high elasticity of revenue to investment, summarized by $\varepsilon = 0.6$. The second case, in Figure 1.b is one of low technology intensity, which implies a lower elasticity of revenue to investment $\varepsilon = 0.1$, for the same elasticity of substitution. Concentration in the lower tale is then larger in the case of Figure 1.a as $\alpha > b$. The figure shows that in the case of high technology intensity, a reduction of variable trade cost from $\tau = 2$ to around $\tau = 1.6$ will lead to exporter productivity gains: the RHS of (36) is above its LHS. However, further reductions of variable trade costs will reduce productivity of exporting firms (i.e. the loss in domestic profits are there too high). For $\varepsilon = 0.75$ (hence $\beta = 0.19$) this crossing point would appear around $\tau = 1.91$. This is not the case in Figure 1.b, where technology intensity is low, so that the RHS of (36) is above its LHS for a significantly wider range of variable trade costs reduction.

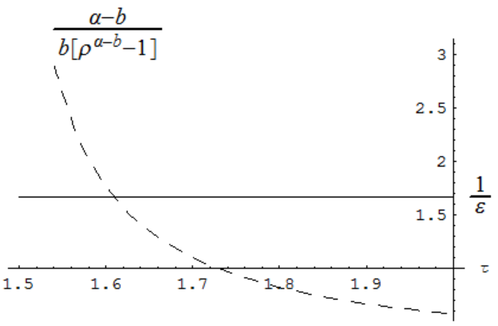

Figure 1.a

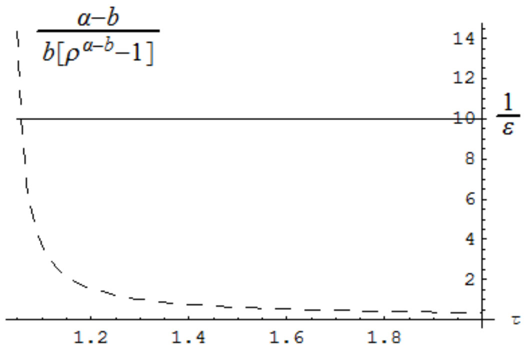

Figure 1.b

We saw in the more general analysis that, in spite of microeconomic adjustments, aggregate productivity improvement might still be ensured by high selection. With a distribution increasingly concentrated at the left, this could be a natural outcome. It is then not surprising that industry-level productivity improvement may arise with Pareto parametrization under rather weak conditions. Average potential efficiency writes $\widetilde{\varphi}_0^t = \varphi_0^* \left[\left[\frac{\alpha}{\alpha-b}\right] \frac{1+n\rho^{-\alpha}\frac{f_x}{f}}{1+n\rho^{-\alpha}} \right]^{\frac{1}{b}}$, so that $\widetilde{\varphi}_0^t$ is unitary elastic with respect to φ_0^* in this particular case. The sufficient condition (33) for industry-level productivity can then be stated as

$$\frac{\frac{d\widetilde{\varphi}_0^t}{d\tau}\frac{\tau}{\widetilde{\varphi}_0^t}}{\frac{d\varphi_0^*}{d\tau}\frac{\tau}{\varphi_0^*}} = 1 + \frac{\alpha n\rho^{-\alpha}}{b\left[1+n\rho^{-\alpha}\frac{f_x}{f}\right]}\frac{\frac{d\rho}{d\tau}\frac{\tau}{\rho}}{\underbrace{\frac{d\varphi_0^*}{d\tau}\frac{\tau}{\varphi_0^*}}_{<0}}\left\{\frac{1-\frac{f_x}{f}}{1+n\rho^{-\alpha}}\right\} < \varepsilon$$

which can be verified with the usual assumption $f_x > f$ as a sufficient (although not necessary) condition. In the parameter setting of Figure 1, this requirement is fulfilled. Hence, in the case of Figure 1.a when τ is small and hence *all* firms are less productive after trade liberalization, one still can observe productivity gains at the aggregate level. These stems from a massive exit of firms whose productivity is reduced by trade liberalization itself. Note that ρ is substantially lower in the case of Figure1.a, especially for low τ. The thresholds φ_0^* and φ_{0x}^* are then closer. This is of course due to reduction of φ_{0x}^*, but also to the level of φ_0^*, since β is high (cf. Proposition 2). As the distribution is highly concentrated at the left (i.e. α is high) the exit of firms is expected to be important.

5 Empirical Discussion

In our model, trade liberalization affects the incentives to undertake productivity-enhancing investments by modifying demand conditions and so profitability. On the one hand, it leads to an expansion of export opportunities through an improvement in foreign market access, which increases expected export profits. On the other hand, profits stemming from domestic sales are negatively affected because of the arrival of new imported varieties at better prices and because of the general-equilibrium consequences of new export opportunities, reflected in the increase of the real wage (the fall in P).[21]

Trade liberalization may also affect productivity by facilitating the access to leading technologies or high-quality embodied in imported inputs. We have ruled out this channel by construction in our analysis in order to focus on trade costs affecting final goods, but it should be part of the vector of controls in an empirical test.[22] Hence a suitable confrontation of our model with the evidence calls for a clear separation of different channels. This is not an easy empirical task as one would need trade cost data on inputs, output and foreign market access, with all together appearing in regressions and hopefully with significant associated coefficients. Those ideal regressions should also test for heterogeneous parameter estimates, at least in terms of export status in order to capture the differentiated effect appearing in our model.

To the best of our knowledge such a kind of test has not been performed yet, presumably because empirical works have focused so far on unilateral trade liberalization episodes or due to the heavy data requirements in the case of bilateral trade agreements. Nevertheless, some pieces of evidence can be collected from the existing studies

[21] Rigorously speaking, this latter mechanism is also at work in the foreign market but is more than offset by the profit gains generated by the reduction in the after-tariff price. It is in fact the increase in export-market profits that allows for an increase in the average profit, which renders the equilibrium consistent with the free-entry condition when selection increases (i.e. when the probability of successful entry decreases).

[22] One way to include this in our model is to assume that the technology input is imported from the rest of the world and its price affected by trade costs. Such a modification would not affect the equilibrium cut-off (price of the technology does not participate in the equilibrium relationship) but can have consequences on firm-level productivity.

and confronted to the mechanisms of our model. We do so in what follows.

The channel linked to the expansion of export opportunities has actually been well documented. Trefler (2004) and Lileeva and Trefler (2010) show that better US market access obtained by Canadian firms through the Canada-US Free Trade Agreement (FTA) has improved Canadian firm's productivity, especially for new-exporters through investments. These new-exporters have also increased their domestic market shares relative to non-exporters. Bas and Ledezma (2010) also find evidence on this line for Chile during the 1982-1999 period with gravity-based estimates of trade barriers (used to circumvent the lack of cross-section variability of Chile's trade policy): relative to non-traded sectors, the reduction of export and import barriers improve plant-level productivity in export-oriented sectors. Bustos (2011) shows that Brazil tariff cuts generate export opportunities for Argentinean firms to enter the export market and expand their R&D investments. All these findings are in line with the first above-mentioned prediction of our model.

Recent empirical micro-level studies also provide some support for the second mechanism, which highlights a potential negative link between the fall in variable trade costs and the incentives to invest. In these studies tariff cuts appear to be associated with the reductions of firms' domestic sales and their capital investments. Baldwin and Gu (2009)' findings suggest that the Canada-US FTA have reduced firms' domestic production and the number of domestic products for non-exporting Canadian plants. Using plant-level data from the Annual Industrial Survey of Mexico during the 1984-1990 period, the Mexican episode of trade liberalization, Kandilov and Leblebicioglu (2012) find that final goods tariffs cuts reduce firms' capital investments. Similarly, Bas and Ledezma (2013) also report similar consequences on sales and investment with firm-level data from India during 1998-2006, a sample period that exploits the second wave of Indian liberalization reforms - the so-called "Ninth-Five-years-plan". Output tariff reductions, after this trade liberalization process, are associated to a contraction of firms' domestic sales and capital investments. This evidence is consistent with our model mechanisms whereby freer trade, by reducing domestic sales and profits, diminishes part of the incentives to undertake technology investments

Influential papers report firm-level productivity gains stemming from unilateral trade liberalization episodes, mainly in developing countries (e.g. Pavcnik 2002, Fernandes, 2007).[23] These, however, are average estimates of the impact of trade liberalization from a unilateral point of view and generally focused on the output side. Recent studies have included in the same specification both tariff on intermediate inputs and on final goods (Schor, 2004, Amiti and Konings, 2007 and Topavola and Khandelwal, 2011) with the aim of differentiating the channels through which trade liberalization affects firm productivity. This type of estimation shows that input tariff cuts have a significantly larger impact on productivity than output tariff reductions. The latter effect typically loses significance and especially magnitude and may even change sign without systematic robustness (Schor, 2004). The afore-cited work of Bas and Ledezma (2010) that controls for export barriers show that import barrier reductions can be associated with a reduction of plant productivity for plants producing in import-competing industries in Chile. Our model actually allows for the possibility of positive effects on average under certain conditions and at

[23] These two references are examples of studies that offer relatively comparable methods. Tybout (2000) review previous evidence linking openness and productivity in developing countries and specially their caveats, p. 34-38.

the same time predicts a negative impact of trade liberalization on non-exporters productivity.[24]

Overall, the evidence offered by the empirical literature tends to be consistent with the idea that trade liberalization, thanks to the expansion of export market opportunities, has a positive effect on productivity improvements of exporting firms, while it reduces domestic sales and investment of non-exporting ones.

6 Conclusion

We have developed an extension of the Melitz (2003) model including an additional stage of technology investment that endogenously determines firm productivity. We have done so within an analytical tractable framework with closed form solutions for the general equilibrium open economy that shows how trade liberalization affect productivity at the firm level. By allowing for a marginally adjustable investment choice, the main contribution of our model is to yield theoretical predictions on the effects of trade variable costs reductions on the magnitude of technology investments. Consequently, heterogenous firms end up reacting differently to investment opportunities and obtain different productivity gains.

The main theoretical findings can be summarized as follows. Two opposite forces affect the relationship between trade liberalization and firm productivity. Variable trade-costs reductions increase expected export profits creating new incentives to firms to upgrade their technology through investments. At the same time, trade liberalization dampens those incentives for technology investments since it reinforces firm selection into the domestic market reducing domestic market shares and post-investment profits. The first positive effect only concerns exporting firms, while the second effect affects all active firms. Trade liberalization will then unambiguously trim down firm productivity of non-exporters, while the effect on exporters depends on parameters. Namely, exporters' and industry-level productivity improvements are likely to pertain in the case where concentration is considerably higher the lower the level of initial efficiency.

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