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Testing convergence in non-stationary panel: a third generation approach

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Résumé

Nous proposons ici une modification de la procédure de test de convergence en panel d'Evans et Karras (1996). La littérature empirique sur la convergence économique a tendance à ignorer les phénomènes d'interdépendances et de changement structurel qui affectent l'équation de convergence alors que la théorie économique insiste beaucoup sur la nécessité de leurs prises en compte. Se référant au test de racine unitaire de Moon et Perron (2004) récemment amélioré par Bai et Ng (2010), nous avons développé une démarche empirique de test de convergence qui permet de purger efficacement les dépendances inter-économies sur la base de la procédure *"Panel Analysis of Nonstationarity in the Idiosyncratic and Common components"* (PANIC) de Bai et Ng (2004). Il a également été montré par simulations de Monte-Carlo qu'en plus de contrôler les interdépendances, PANIC offre aussi l'avantage de traiter naturellement l'existence d'un unique changement structurel et d'éliminer les problèmes de puissance de test qu'il engendre. Des applications sont ensuite menées sur un échantillon de 40 pays riches et pauvres dont la moitié est membre de l'OCDE et l'autre moitié composée de pays de l'Afrique subsaharienne.

Mots clés : β-convergence ; données de panel ; racine unitaire ; modèle factoriel ; dépendance interindividuelle ; changement structurel.

Abstract

This paper proposes a modification of Evans and Karras' (1996) testing procedure for economic convergence in panel. While economic literature argues in favour of the presence of cross-unit dependencies and breaks in macro-economic time series, convergence tests in panel generally rule out theses phenomenon. Referring to panel unit root test proposed by Moon and Perron (2004) recently improved by Bai and Ng (2010), our empirical procedure allows for purging effects of cross-country correlation and structural instability from the convergence equation on the basis of the Panel *Analysis of Nonstationarity in the Idiosyncratic and Common components* (PANIC). Using an extension of Bai and Ng's (2010) simulations by including structural break, we show that in addition of controlling correlations, PANIC also offers the advantage of naturally treating the presence of a single structural change and then permits to solve problems of low power it generates. Applications are then conducted using sample composed by 20 countries of OECD members and 20 countries in Sub-Saharan Africa.

Keywords: β-convergence; Unit root; Panel data; Factor model; Cross-sectional dependence; Structural change.

Classification JEL : C22; C23; O40; R11.

TESTING CONVERGENCE IN NON-STATIONARY PANEL: A THIRD GENERATION APPROACH *

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This work is dedicated to the memory of Marcellin EDJO, who disappeared prematurely on 14 December 2008 in Dakar (Senegal). Marcellin was an economist at the BCEAO. He worked on economic convergence in the context of properties of non-stationary series. He contributed to a first version of this paper.

Introduction

Since the work by Baumol (1986) and by Barro and Sala-i-Martin (1991, 1995), many papers have set about analyzing convergence using two conventional approaches: $\beta-$convergence and σ-convergence. These two forms of convergence have many applications in time series properties. Indeed, the development of econometric analysis techniques and the availability of databases (Summers and Heston, 1991) covering large periods provide the opportunity to go beyond cross-sectional analysis and to exploit the properties of non-stationary time series (Bernard and Durlauf 1995; Edjo 2003) so as to better inform the debate on economic convergence.

Convergence tests have also expanded within the framework of panel data analysis. The first panel data tests were based primarily on the methodology used in cross-sectional analysis (e.g. Islam, 1995; Berthelemy et al., 1997). Then, just as with individual time series, panel unit root tests were used to study economic convergence. This procedure based on panel unit root tests was first implemented by Quah (1992), Evans (1996), Evans and Karras (1996), Bernard and Jones (1996), and Gaulier et al. (1999) among others. More powerful tests were devised by combining the cross-section and time dimensions. Until now, two generations of unit root tests have been distinguished. Most methods of analyzing economic convergence using the properties of non-stationary series refer to the first generation, which assumes independence between individuals (Harris and Tzavalis, 1999; Maddala and Wu, 1999; Hadri, 2000; Choi, 2001; Levin et al., 2002; Im

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et al., 2003). However, as Hurlin and Mignon (2005) pointed out, this assumption of cross-section independence is particularly troublesome in applications of macro-economic convergence tests. The second generation of unit root tests is generally based on common factor models (Bai and Ng, 2004; Moon and Perron, 2004; Pesaran, 2007; and Bai and Ng, 2010) and takes into account more general forms of cross-sectional dependencies.

The empirical procedure we propose here is inspired by the second generation of unit root tests based on factor models and explicitly takes account of the dependencies in the individual dimension. We focus on the fact that the cross-country correlation in the convergence equation is due not only to simple correlation of residuals but also to the presence of one or more common factors that jointly affect the real per capita GDP. Therefore, the study of the convergence in panels, based on the standard ADF model as suggested by Evans and Karras (1996), is no longer suitable because it leads to tests with very low power (Strauss and Yigit, 2003).

Another issue addressed in this procedure is the existence of breaks in per capita GDP. Studies of structural change in panel data with cross-sectional dependence are very rare (Bai and Carrion-i-Silvestre, 2009; Carrion-i-Silvestre and German-Soto, 2009). As pointed out by Carrion-i-Silvestre et al. (2005), ignoring these shocks in the econometrics of panel data can lead to biases that lead to wrong conclusions. Financial and economic crises, economic reforms, and so on are events that may cause such shocks.

In the next section we present the approaches generally used to test for convergence in non-stationary panel data by focusing on Evans and Karras' procedure (1996). Then in section 2 we apply the procedure we propose and which is inspired by these traditional approaches. In section 3 we conduct a Monte-Carlo simulation to analyze the impact of the proposed procedure on performance tests. Section 4 presents an application using a sample of OECD member countries and a sample of countries in Sub-Saharan Africa.

1 Convergence tests in panel data econometrics

Convergence tests in panel data are generally based on the standard approach in cross-section the purpose of which is to test whether economies with low initial income relative to their long-term position will grow faster than economies with high initial income. This involves applying ordinary least squares (OLS) to the equation

$$\frac{1}{T}\ln(y_{i,T}/y_{i,0}) = \kappa + \beta\ln(y_{i,0}) + \varphi\Xi_i + \xi_i \qquad \xi_i \backsim i.i.d(0,\sigma_\xi^2) \tag{1}$$

where y_i is real per capita GDP of country i, Ξ_i is a vector of controlled variables so as to maintain a constant steady state of each economy; i, and ξ_i is the error term. The index T refers to the length of the time interval. κ, β and φ are unknown parameters which have to be estimated. The convergence speed $\theta = -\ln(1+\beta T)/T$ is the rate at which a given economy catches up to its steady-state. The null hypothesis tested is the lack of convergence against the alternative that some countries converge to a certain level of production that is initially different. If the estimated coefficient β is negative and significant, the hypothesis of convergence can be accepted. This means that once the variables influencing growth are controlled, low-income economies tend to grow faster towards their own steady state. With the coefficient β, it is possible to deduce the time necessary for countries to make up half the gap separating them from their steady state. This half-life is given by the expression $\tau = -\ln(2)/\ln(1+\beta)$.

However, OLS estimation of (1) is useful for inference under certain conditions only. Evans and Karras (1996) explain that the estimators $\hat{\beta}$ and $\hat{\varphi}$ obtained by applying ordinary least squares to (1) are valid only if ξ_i and $y_{i,0}$ are uncorrelated and if the constant term is generated as follows

$$\kappa_i = \psi^{'}\Xi_i \tag{2}$$

with $\psi \equiv (\lambda-1)\varphi/\beta$. [1] In panel data, Evans and Karras' procedure (1996) based on unit root tests is a basic procedure for many studies of economic convergence tests (see for example Gaulier et al., 1999). Considering a group of N countries, these authors show that the countries converge if deviations of the log per capita GDP from the international average are stationary for each country. Let y_{it} be the log per capita GDP of country i at the period t with $i = 1, ...N$; $t = 1, ...T$, and $\bar{y}_t$ be the international average[2] of y_{it}. This is to test whether the data generating process $(y_{it} - \bar{y}_t)$ is stationary for all i

$$\lim_{h\to\infty}(y_{i,t+h} - \bar{y}_{t+h}) = \mu_i. \tag{3}$$

Convergence occurs if for each i deviations of per capita GDP from the international average tend to a constant when $t \to \infty$. Specifically, the convergence hypothesis is accepted only if $y_{it} - \bar{y}_t$ are stationary while the y_{it} are integrated of order 1. In such a case, we have stochastic convergence. However, as stressed by Carrion-i-Silvestre and German-Soto (2009), stochastic convergence is a necessary but not a sufficient condition to satisfy the definition of β-convergence. With $y^c_{it} = y_{it} - \bar{y}_t$, the data generating process proposed by Evans (1996) is

$$y^c_{it} = \kappa_i + \lambda y^c_{i,t-1} + u_{it} \tag{4}$$

where $\lambda \equiv (1+\beta T)^{(1/T)}$ is less than 1 if the N economies converge and in this case $\beta \prec 0$. However, there is divergence if $\lambda = 1$ which implies that $\beta = 0$. The constant term κ_i is specific to each country and the error term is serially uncorrelated. Evans and Karras (1996) further show that in the case where the errors are correlated in the individual dimension, this specification entails serious problems of statistical inference. International trade in goods and assets means that innovations are probably correlated. In addition, given the specificity of countries in terms of technology, the parameter λ should be specific to each economy. Therefore, the ADF specification in panels with a heterogeneous autoregressive root is generally used as an alternative

$$\Delta y^c_{it} = \kappa_i + \rho_i y^c_{it} + \sum_{s=1}^{p}\gamma_{i,s}\Delta y^c_{i,t-p} + u_{it}. \tag{5}$$

[1] This equation shows the link between the cross-section and panel specifications. λ is a convergence parameter that we will define in the next section.

[2] $\bar{y}_t = \sum_{i=1}^{N} y_{it}/N$.

The parameter ρ_i is negative if the economies converge and is zero if they diverge. The roots of $\sum_s \gamma_{i,s} L^s$ are outside the unit circle. In the procedure proposed below, we use a general specification of equation (4) which allows better control of cross-sectional and serial correlations of u_{it}. It also takes into account a possible structural change in the mean of the data generating process.

2 An alternative procedure

This section presents the proposed empirical approach. By using PANIC (*Panel Analysis of Nonstationarity in the Idiosyncratic and Common components*), we test first for stochastic convergence, a primary condition of β-convergence on the basis of Bai and Ng's statistics (2010). This is to test non-stationarity of per capita GDP cross economies differences ($H_0 : \lambda = 1$). If stochastic convergence is confirmed, we then move on to conduct the β-convergence test.

2.1 Econometric specification

As mentioned previously, specification (4) is useful only under certain conditions and if they do not pertain it will be very challenging to obtain a consistent estimate of parameters. These conditions are relative to the error term u_{it} and can be summarized in two general points related by Evans (1996). (i) u_{it} is a serially uncorrelated error term with a zero mean and finite and constant variance. (ii) also, u_{it} is contemporaneously uncorrelated across countries. To deal with cross-sectional correlation of u_{it} we use the data generating process of Moon and Perron (2004) to define a general form of equation (4)

$$y_{it}^c = \kappa_i + \lambda_i y_{i,t-1}^c + u_{it}. \tag{6}$$

In this model, the correlations among the cross-sectional units of u_{it} are captured by using a factor model

$$u_{it} = \pi'_i f_t + \varepsilon_{it} \tag{7}$$

where f_t is a $(T \times r)$ matrix representing the common factors, π_i is a $(r \times 1)$ vector of factors loadings and ε_{it} represents a $(T \times 1)$ vector of idiosyncratic error. If, following Bai et Ng (2010), we admit that the common and the idiosyncratic components have the same order of integration and that $\lambda_i = \lambda$ for all i, we can write

$$y_{it}^c = \kappa_i + \pi'_i F_t + e_{it} \tag{8}$$

where $F_t - \lambda F_{t-1} = f_t$ and $e_{it} - \lambda e_{i,t-1} = \varepsilon_{it}$. The transition to specification (8) makes it possible to estimate λ without being confronted with problems caused by failure to fulfill conditions (i) and (ii). We first deal with these cross-sectional dependencies by using PANIC to remove common factors. Then the null hypothesis of divergence is tested on the de-factored variable x_{it}. This is equivalent to testing the null hypothesis of unit root $H_0 : \lambda_i = 1 \ \forall i$ against the alternative hypothesis of stationarity $H_1 : \lambda_i \prec 1$ for some individuals in the panel.

The problems of structural changes that may affect the economies are also taken into account in this procedure. Paci and Pigliaru (1997) argued that structural change plays a fundamental role in the convergence process. It is closely associated with shifts of resources in different sectors. Thus, just as with economic interdependence, the omission of these breaks in the modeling convergence process generally leads to the hypothesis of convergence being wrongly rejected. To take this into account, we propose another general form of equation (8) which admits the occurrence of a single break in the mean

$$y_{it}^c = \kappa_i + \theta_i DU_{i,t} + \pi'_i F_t + e_{it} \tag{9}$$

where $DU_{i,t} = 1$ for $t \succ T_{b,i}$ and 0 elsewhere. $T_{b,i}$ denotes the break in the intercept for the $i-th$ individual. The first-differenced form of equation (9) is

$$\Delta y_{it}^c = \theta_i D(T_{b,i})_t + \pi'_i \Delta F_t + \Delta e_{it} \tag{10}$$

where $D(T_{b,i})$ are impulses such that $D(T_{b,i})_t = 1$ for $t = T_{b,i} + 1$ and 0 elsewhere. Following Bai and Carrion-i-Silvestre (2009)[3], we ignore these impulses since they take into account a few unusual events and their effect is asymptotically negligible. Let us define $\hat{F}_t = \sum_{s=2}^{t} \Delta F_t$ and $\hat{e}_{it} = \sum_{s=2}^{t} \Delta e_{is}$. Since the effects of $\theta_i D(T_{b,i})_t$ are negligible and can be included in the idiosyncratic error, we have for $t = 2, \ldots, T$,

$$\hat{y}_{it}^c = \pi'_i \hat{F}_t + \hat{e}_{it}. \tag{11}$$

The $\hat{y}_{it}^c$ series preserve the same non-stationarity property as the original series y_{it}^c (Bai and Ng, 2004). In addition, model (11) has two important advantages. Firstly, this process is not affected by structural change. Thus, we face the simple case of a test without a break. Second, unlike Moon and Perron's approach (2004) which uses an orthogonalization procedure *à la* Philips and Sul (2003) to eliminate the common factors, these factors are estimated explicitly before being eliminated from the model. As pointed out by Bai and Ng (2010), the method presented by Moon and Perron (2004) to eliminate common and deterministic components causes serious problems of power especially when the model contains a trend. In section 4 we make Monte-Carlo simulations to verify whether this procedure can affect the test performances. Let $\hat{x}_{it}$ be the de-factored form of $\hat{y}_{it}^c$. Since $\hat{x}_{it} = \hat{e}_{it} = \lambda \hat{e}_{i,t-1} + \hat{\varepsilon}_{it}$, then the de-factored model is

$$\hat{x}_{it} = \lambda \hat{x}_{i,t-1} + \hat{\varepsilon}_{it} \tag{12}$$

where $\hat{\varepsilon}_{it}$ is uncorrelated across country accordance to condition (ii). Note that to select the number of common factors r we use criteria developed by Bai and Ng (2002).

[3] Bai and Carrion-i-Silvestre (2009) adopt this procedure in their modified Sargan-Bhargava (MSB) tests which takes into account multiple structural changes and common factors.

2.2 Testing for stochastic convergence

In the previous section we see that the implementation of the procedure requires the selection of the number of common factors to be eliminated from the data generating process in order to define a consistent estimate of λ. In this section, we present the method used to estimate r, the technique for the estimation of λ, and the test statistics of the null hypothesis $\lambda = 1$.

2.2.1 Selection and estimation of common factors

The matrix of estimated factors in first difference noted $\Delta\tilde{F}$ is equal to $\sqrt{T-1}$ times the eigenvectors corresponding to the r largest eigenvalues of the $(T-1) \times (T-1)$ matrix $y^c y^{c\prime}$. Considering the normalizations $\tilde{\pi}'\tilde{\pi}/N = I_r$ and $\Delta\tilde{F}'\Delta\tilde{F}/(T-1) = I_r$, the matrix π of factor loadings can be obtained by ordinary least squares $\tilde{\pi}' = \left(\Delta\tilde{F}'\,\Delta\tilde{F}\right)^{-1}\Delta\tilde{F}'\,y^c = \Delta\tilde{F}'\,y^c/(T-1)$. Furthermore, to estimate r we use the IC_1 and BIC_3 information criteria (Bai and Ng 2002). The BIC_3 criterion is a modification of the usual BIC which perform better in small samples ($N \leq 20$). Let $V(r, \Delta F)$ be the sum of squared residuals (divided by $N(T-1)$)[4] of the regression of $\hat{y}^c_{it}$ on the r factors for each i. For panel with moderate N as in the case of macroeconomic convergence analysis, we can use $V(r, \Delta F) + r_{\max} g_{BIC}(N,T)$ [5] where $g_{BIC}(N,T)$ is the penalty function. In this case, r can be estimated consistently with $g_{BIC}(N,T) = \frac{(N+T-r)ln(NT)}{NT}$ by minimizing

$$BIC_3(r) = V(r, \Delta\tilde{F}) + r\widehat{\sigma}^2_e(r_{\max})\left(\frac{(N+T-r)\ln(NT)}{NT}\right). \tag{13}$$

Where $\hat{\sigma}^2_e$ is the variance of the estimated idiosyncratic component. For IC_1 the penalty function is $g_{IC}(N,T) = \frac{N+T}{NT}\ln\left(\frac{NT}{N+T}\right)$ and the problem consists in minimizing

$$IC_1(r) = \ln\left(V(r, \Delta\tilde{F})\right) + r\widehat{\sigma}^2_e(r_{\max})\frac{N+T}{NT}\ln\left(\frac{NT}{N+T}\right). \tag{14}$$

In the next section, we use this approach inspired by PANIC to define a pooled estimator of λ noted $\hat{\lambda}^*$ and then we construct the test statistics using Bai and Ng's (2010) procedure.

2.2.2 Estimation of λ and construction of test statistics

The test statistics of the null hypothesis $\lambda = 1$ can be constructed from the pooled modified OLS estimator of the autoregressive root. This estimator is corrected to take into account the condition (i). Thus, the possible serial correlations of the residuals $\hat{\varepsilon}_{it}$ are controlled. Let $\hat{\phi}_\varepsilon$ be the sum of positive autocovariances of the errors and $\hat{x}$ the $(T-2) \times N$ matrix of $\hat{x}_{it}$. The modified OLS estimator is

$$\hat{\lambda}^* = \frac{trace(\hat{x}'_{-1}\hat{x}) - NT\hat{\phi}_\varepsilon}{trace(\hat{x}'_{-1}\hat{x}_{-1})}. \tag{15}$$

[4] $V(r, \Delta F)$ is the variance of the idiosyncratic component estimated with the maximum number of factors.

[5] r_{max} is the maximum number of factors.

Following Bai and Ng (2010), two test statistics of the null hypothesis $\lambda = 1$ are constructed using this estimator of the autoregressive root. The statistics are noted P_a and P_b and are the analogs of t_a and t_b of Moon and Perron (2004). Both follow a standard normal law and we have

$$P_a = \frac{T\sqrt{N}\left(\hat{\lambda}^* - 1\right)}{\sqrt{2\hat{\nu}_\varepsilon^4/\hat{\omega}_\varepsilon^4}} \rightarrow N(0,1); \tag{16}$$

$$P_b = T\sqrt{N}\left(\hat{\lambda}^* - 1\right)\sqrt{\frac{1}{NT^2} trace\left(\hat{x}'_{-1}\hat{x}_{-1}\right)\frac{\hat{\omega}_\varepsilon^2}{\hat{\nu}_\varepsilon^4}} \rightarrow N(0,1) \tag{17}$$

where ω_ε^2 and ν_ε^4 respectively correspond to the means on N of the individual long-term variances $\omega_{\varepsilon,i}^2$ and of squared individual long-term variances $\phi_{\varepsilon,i}^4$ of ε_{it}. Let $\hat{\Gamma}_i(j)$ be the residual empirical autocovariance, we have

$$\hat{\Gamma}_i(j) = \frac{1}{T}\sum_{t=1}^{T-j}\hat{\varepsilon}_{i,t}\hat{\varepsilon}_{i,t+j}.$$

From $\hat{\Gamma}_i(j)$, it is possible to construct an estimator of the individual long-term variances[6]

$$\hat{\omega}_{\hat{\varepsilon},i}^2 = \frac{1}{N}\sum_{j=-T+1}^{T-2}\omega(q_i,j)\hat{\Gamma}_i(j); \qquad \hat{\phi}_{\hat{\varepsilon},i} = \sum_{j=1}^{T-1}\omega(q_i,j)\hat{\Gamma}_i(j).$$

These individual variances are used to define the estimates of the means of the individual long-term variances as follows

$$\hat{\omega}_{\hat{\varepsilon}}^2 = \frac{1}{N}\sum_{i=1}^{N}\hat{\omega}_{\hat{\varepsilon},i}^2; \qquad \hat{\phi}_{\hat{\varepsilon}} = \frac{1}{N}\sum_{i=1}^{N}\hat{\phi}_{\hat{\varepsilon},i}; \qquad \hat{\nu}_{\hat{\varepsilon}}^4 = \frac{1}{N}\sum_{i=1}^{N}(\hat{\omega}_{\hat{\varepsilon},i}^2)^2.$$

The test statistics are obtained by substituting the estimated values of these variances in the expressions of P_a and P_b. If the realization of the statistic $P_{a,b}$ is lower than the normal critical level, we accept the hypothesis of stochastic convergence.

2.3 Analyzing β-convergence

This subsection presents the method used to analyze β-convergence when the hypothesis of stochastic convergence is accepted. The aim is to estimate the implied value of β given by $\hat{\beta} = ((\hat{\lambda}^*)^T - 1)/T$ in order to analyze β-convergence. For this purpose we use $\hat{\lambda}^*$, the consistent estimator of λ. The procedure is summarized in three steps for estimating λ and testing the null hypothesis $\lambda = 0$.

Step 1: We apply PANIC to equation (8) and obtain the de-factored model

$$\hat{x}_{it} = \lambda\hat{x}_{i,t-1} + \hat{\varepsilon}_{it}$$

[6] $q_i = 1.3221\left[\frac{4\hat{\psi}_{i,1}^2 T_i}{(1-\hat{\psi}_{i,1})^4}\right]^{1/5}$ with $\hat{\psi}_{i,1}$ the estimator of the first order autocorrelation of $\hat{\varepsilon}_{it}$; $\omega(q_i,j) = \frac{25}{12\pi^2 w^2}\left[\frac{\sin(6\pi w/5)}{6\pi w/5} - \cos\left(\frac{6\pi w}{5}\right)\right]$ with $w = \frac{j}{q_i}$.

where the variables are defined as in equation (12). Then, for each i, the $\hat{x}_{it}$ series are normalized by the OLS regression standard error $\hat{\sigma}_{\hat{\varepsilon}_i}$ to control for heterogeneity across countries. The normalized series is

$$\hat{z}_{it} = \hat{x}_{it}/\hat{\sigma}_{\hat{\varepsilon}_i}.$$

Step 2: In this step we construct the following normalized model to estimate λ

$$\hat{z}_{it} = \lambda \hat{z}_{i,t-1} + \hat{v}_{it} \tag{18}$$

where $\hat{v}_{it} = \hat{\varepsilon}_{it}/\hat{\sigma}_{\hat{\varepsilon}_i}$. Let $\hat{z}$ the matrix of observations $\hat{z}_{it}$ and $\hat{z}_{-1}$ the matrix of lagged observations. The corrected estimator of λ is

$$\hat{\lambda}^* = \frac{trace(\hat{z}'_{-1}\hat{z}) - NT\hat{\phi}_\varepsilon}{trace(\hat{z}'_{-1}\hat{z}_{-1})}.$$

Step 3: On the basis of the modified pooled estimator of the normalized equation, we define the corrected t-statistic

$$t^*(\lambda) = \frac{\hat{\lambda}^*}{\hat{\sigma}_{\lambda^*}},$$

where

$$\hat{\sigma}_{\lambda^*} = \hat{\sigma}_{\hat{v}} \left(\sum_{i=1}^{N} \sum_{t=2}^{T} \hat{z}_{i,t-1}^2 \right)^{-1/2} \quad ; \quad \hat{\sigma}_{\hat{v}} = \sqrt{trace((\hat{z} - \hat{\lambda}^*\hat{z}_{-1})(\hat{z} - \hat{\lambda}^*\hat{z}_{-1})')/NT}.$$

We compare this statistic with the appropriate critical value to test the null hypothesis $\lambda = 0$. If the null is accepted, we replace $\hat{\lambda}$ by 0 in the expression of $\hat{\beta}$ given earlier. Otherwise, the estimated value of $\hat{\lambda}$ obtained in Step 2 will be used.

3 Simulations

This section presents the results of Monte-Carlo simulations whose purpose is to check whether the use of the PANIC procedure can also help to eliminate statistical incidences of structural change problems. The verification of this hypothesis is important in the sense that it implies that the specification (18) can be used not only to test the hypothesis of convergence without being confronted with problems of correlations in the error term but in addition it can handle breaks affecting the mean of the series. We show that even in the presence of a single break, the test statistics P_a and P_b can be used without any negative impact on the size and power of the test. We conduct two experiments[7] using MATLAB 6.5. In *Experiment 1*, the data generating process is the same as in Bai and Ng (2010) while in *Experiment 2* this model is augmented by a break in the mean. However, we only consider the case of a single common factor in which common shocks are $i.i.d$, $(F_{tj}, \pi_{ij}, e_{it}) \sim \ iidN(0, I_3)$.

[7] Program realized by the authors. We thank Christophe Hurlin, Serena Ng and Pierre Perron for making available the additional codes used to develop programs of both experiments.

Experiment 1:

$$y_{it}^{c} = \kappa_i + \sum_{j=1}^{r} \pi_{ij} F_{tj} + e_{it}$$

where $F_t = \Phi F_{t-1} + \eta_t$ and $e_{it} = \lambda e_{i,t-1} + \varepsilon_{it}$.

In *Experiment 2* we include break points which are randomly positioned for each i with break fractions $\alpha_i = T_{b,i}/T$ following $\alpha_i \sim \ U[0.2, 0.8]$

Experiment 2:

$$y_{it}^{c} = \kappa_i + \theta_i DU_{i,t} + \sum_{j=1}^{r} \pi_{ij} F_{tj} + e_{it}$$

where $F_t = \Phi F_{t-1} + \eta_t$ and $e_{it} = \lambda e_{i,t-1} + \varepsilon_{it}$.

In both experiments $\kappa_i \sim \ N(0, 1)$. The PANIC procedure is used in both cases to define the de-factored form of the model

$$\hat{x}_{it} = \lambda \hat{x}_{i,t-1} + \hat{\varepsilon}_{it}.$$

To study the size of the test we have, following Bai and Ng (2010), $\lambda_i = \Phi = 1$ for all i. For power, we have considered values of λ_i that are not far from the null hypothesis of unit root. Thus, under the alternative, the parameter λ is specific to each individual with $\lambda \sim \ U[0.9, 0.99]$, whereas $\Phi = 0.5$. The number of common factors is estimated using BIC_3 and IC_1 criteria. Following Bai and Ng (2002, 2004, 2010) and Moon and Perron (2004), the maximum number of factors is set to 8. Simulations are conducted using 5,000 replications with $N = \{20, 50\}$ and $T = \{50, 100\}$, and we consider the 5% significance level.

[TABLE 1 HERE]

Table 1 presents results for power and size in each empirical experiment described above. For these two data generating processes, the properties of size and power of P_a and P_b tests are studied by considering the percentage of replications in which the unit root hypothesis is rejected. This table also shows the average number of factors estimated using the selection criteria and the average true number of factors which is equal to 1. As expected, the results show that in the presence of a single break, the finite-sample properties of P_a and P_b tests are not affected when PANIC is used. The properties of size and power of the two experiments are very similar. In addition, estimation of the number of common factors is robust to a single break. The average number of common factors remains the same for both experiments and irrespective of the couple (N, T) considered.

4 Application

4.1 Data

The data are from the World Development Indicators (WDI) of World Bank Group. These are annual real per capita GDP covering the period 1975-2008. To compare results for developed and poor countries we consider two samples. The first sample *OECD* includes 20 OECD member countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, United Kingdom, United States. The second sample called *CFA* comprises 13 CFA zone member countries with 7 countries in Sub-Saharan Africa. The 13 member countries are Benin, Burkina Faso, Cameroon, Central African Republic, Chad, Congo, Côte d'Ivoire, Gabon, Guinea Bissau, Mali, Niger, Senegal and Togo. Generally, these countries have highly correlated business cycles (Diagne and Niang 2008). The 7 countries of the *AFRICA* sample are: Democratic Congo, Gambia, Ghana, Liberia, Nigeria, Sierra Leone, South Africa.

A global sample called *GLOBAL* composed by these two groups is also considered. This last sample comprises 40 countries including poor and rich economies.

4.2 Results

4.2.1 Comparing the results from the different generations of tests

We use several test statistics of the three generations[8] developed in the literature to test the non-stationarity of the deviations of the per capita GDP from the international average. All test results are presented in Table 2 and are based on a data generating process whose deterministic component contains an intercept augmented with a single break if necessary. Using these statistics makes it possible to test the null hypothesis of divergence and to make a comparative study of the results by analyzing the impact of interdependence and/or structural change. Initially, tests for structural change are conducted using Bai and Perron's procedure (1998). The null hypothesis of independence is also verified on the basis of Pesaran's CD statistic (2004), which is robust to breaks. The results of these tests are given in Tables 4 and 5 and show that there are problems of break and cross-section dependence. For the *GLOBAL* sample, the CD test results based on $ADF(p)$ regression residuals are significant at the 5% level for both the log per capita GDP and its mean-centered variant. This also applies to the OECD sample and regardless of the lag order $p = 1, 2, 3$. For the *AFRICA* group, only the test on the log per capita GDP rejects the null hypothesis of independence, the tests applied on the cross-section demeaned log per capita GDP reject the null hypothesis.[9]

For the convergence tests, the test statistics reported in Table 3 are presented in the appendix. Other statistics are also available in the literature but here we consider only those best suited to the structure of our panel given the specific properties of these tests. For the procedures of Im et al. (1997), Maddala and Wu (1999), and Levin et al. (2002), which belong to the first generation

[8]We are grateful to Christophe Hurlin (University of Orleans), Serena Ng (Columbia University) and Pierre Perron (Boston University) for having kindly posted MATLAB programs necessary for the implementation of the first and second generation of tests presented in Table 2. We also thank Josep Lluís Carrion-i-Silvestre (University of Barcelona) and Tomás del Barrio Castro (University of the Balearic Islands) for providing the GAUSS programs used in the tests $stat_{CDL}$ and P_m.

[9]In the application of the proposed procedure, we have nevertheless retained the model (9) for this group of countries.

of tests ignoring the correlations between countries and structural changes, the corresponding statistics W_{tbar} and t_ρ^* are compared to the critical values of the normal distribution, while P_{MW} compared to a threshold of a $\chi^2(2N)$. However, for the 'second generation' tests, which allow cross-section dependencies to be modeled on the basis of a factor model (Bai and Ng, 2004; Moon and Perron, 2004; Pesaran, 2007), the statistics[10] $P^c_{\hat{e},Choi}$ and MQ_c respectively test the non-stationarity of the idiosyncratic and common components from the same country. Contrary to $P^c_{\hat{e},Choi}$ which follows a standard normal law, MQ_c and $CIPS^*$ are nonstandard and their critical values are provided by the authors. At the 5% threshold, these critical values are respectively equal to -57.04 and -2.22. According to the test of Carrion-i-Silvestre et al. (2001) whose model takes into account a single break in the mean and ignores the correlations in the individual dimension, the corresponding statistic follows a normal distribution with zero mean and a variance that depends on T and the position of the break. Finally, the statistics P_m used for the test of Bai and Carrion-i-Silvestre (2009), which includes both interdependence and structural change and thus, belongs to a third generation[11] of tests, here admits a standard normal distribution.

[TABLE 2 HERE]

The results of the first and second generations of tests display significant disparity between the results of the same generation and between those of the two different generations. Among the first tests, only that of Levin et al. (2002) accepts the hypothesis of convergence at the 1% level for *OECD* and *GLOBAL*, and 10% for the sample *AFRICA*. The statistic P_{MW} of Maddala and Wu (1999) validates the hypothesis of convergence for OECD member countries alone with a significance level of 5% while the test W_{tbar} of Im et al. (1997) accepts the hypothesis of convergence for countries in the *GLOBAL* sample and for *OECD* countries at the respective thresholds of 5% and 1%. The hypothesis of convergence is definitely rejected for the African countries.

The inclusion of the cross-section dependencies only (second generation of tests) also leads to mitigated results. With the procedure of Bai and Ng (2004), convergence is rejected regardless of the sample considered. This procedure has the advantage of identifying the source (idiosyncratic or common) of non-convergence between the economies. The lack of convergence among OECD countries and those of the *GLOBAL* sample is caused by common factors. To the extent that most countries considered are active in the same economic or monetary areas, this situation of divergence may seem contradictory. Economic theory, particularly in the area of economic and monetary integration, supports the claim that the economic interdependencies generated by the policies of sub-regional integration should accelerate the convergence process. However, it should be noted that apart from the impact of integration policies, economies are also affected by shocks related to the global economy which, as shown by the test results of Bai and Ng (2004), are real sources of divergence. Bai and Carrion-i-Silvestre (2009) argue that when the data generating process contains common factors, $I(0)$ factors represent the common shocks, while $I(1)$ factors model the

[10] Please note that the pooled statistic of the idiosyncratic component $P^c_{\hat{e},Choi}$ is standardized from the standardization procedure of Choi (2001).

[11] In this work, we call the third generation of tests those which take into account both the economic interdependence and structural change.

effects related to unobservable global stochastic trends. For example, Hurlin and Mignon (2005) note that in the analysis of properties of non-stationarity of GNP series, $I(1)$ common factors can be assimilated to the factors of global growth. Still with regard to the second generation of tests, Moon and Perron's statistics (2004) noted t_b^* accepted the convergence hypothesis at the 1% level for our three samples, completely contradicting the results of the $CIPS^*$ test that conclude in favor of divergence for these samples. In the $CIPS^*$ test, the hypotheses are formulated so that in the alternative, we consider two categories of countries: a first category of converging economies and a second category of countries which diverge. Thus, if the alternative hypothesis is accepted, this reflects the fact that there is at least one country whose per capita GDP converges to the international average. This also applies to the tests of Im et al. (1997) and Moon and Perron (2004). In Table 6, we use the individual CADF statistics of Pesaran (2007) for each economy to identify countries with a per capita GDP which converges to the international average.

In order to study the situation where only the structural change is taken into account, the test of Carrion-i-Silvestre et al. (2001) which is an extension of the unit root test (first generation) developed by Harris and Tzavalis (1999) is also implemented. Here, the estimated break dates are common to all economies and are obtained on the basis of a *Supremum* statistic. The common dates correspond to 1989, 1995 and 1989 respectively for the samples $AFRICA$, $OECD$ and $GLOBAL$. The test results obtained with Carrion-i-Silvestre et al.'s statistics (2001) are identical to those of the $CIPS^*$ tests in the sense that they conclude in favor of the non-stationarity of the cross-section demeaned per capita GDP regardless of the sample.

In general, the finding that emerges through the study of the results of the first two generations of tests is that although considerable progress is being made in the literature on non-stationary panels, the results related to empirical tests of convergence are very mitigated and not always in line with the predictions of economic theory. Thus, it seems essential to go further towards effectively integrating the various phenomena that may affect the convergence equation, the omission of which generally leads to the convergence hypothesis being wrongly rejected. Moreover, the results from the P_m test of Bai and Carrion-i-Silvestre (2009), which belongs to the third generation of tests including economic co-movements and structural change, accept the hypothesis of economic convergence for the three groups of countries at the 1% threshold. The following section presents an application based on the proposed empirical procedure which, in addition to testing the convergence hypothesis by taking into account both interdependencies and breaks, allows us to go further by analyzing the $\beta-$convergence.

4.2.2 Results based on the proposed approach

- **Critical values of the statistics** $t^*\left(\hat{\lambda}\right)$

The analysis of the results obtained in the framework of the proposed approach requires knowledge of the marginal significance level of the corrected t-statistics $t^*\left(\hat{\lambda}\right)$. We implement Monte-Carlo simulations[12] from which the critical values for standard threshold of 1%, 5% and 10% can be determined. The simulation procedure is as follows. First, the parameters (variances) of $\hat{\varepsilon}_{it}$ are collected for each i to construct the null model $\hat{x}_{it} = \hat{\varepsilon}_{it}$. To this end, we first apply the PANIC procedure to the model $y_{it}^c = \kappa_i + \theta_i DU_{i,t} + \lambda'_i F_t + e_{it}$ where $F_t = \Phi F_{t-1} + \eta_t$ and $e_{it} = \lambda e_{i,t-1} + \varepsilon_{it}$. The de-factored and de-trended model $\hat{x}_{it} = \lambda \hat{x}_{i,t-1} + \hat{\varepsilon}_{it}$ is then estimated by OLS to obtain residuals $\hat{\varepsilon}_{it}$. For each i, using the variance of $\hat{\varepsilon}_{it}$, we generate 10,000 data sets[13] for the null model $\hat{x}_{it} = \hat{\varepsilon}_{it}$ with $\hat{\varepsilon}_{it} \sim \ iidN\left(0, \sigma^2_{\hat{\varepsilon}_i}\right)$. Then, on the basis of these data sets of the null model, we estimate the alternative model $\hat{x}_{it} = \lambda \hat{x}_{i,t-1} + \hat{\varepsilon}_{it}$. So, steps 2 and 3 of the procedure presented in sub-section 3.3 are implemented to obtain the unbiased pooled estimator of the normalized model and compute the test statistics $t^*\left(\hat{\lambda}\right)$. With a sample of 10,000 values of $t^*\left(\hat{\lambda}\right)$ we obtain critical values which correspond to quantiles 1%, 5% and 10%. Then, $t^*\left(\hat{\lambda}\right)$ is compared to these critical values.

- **Discussions of results**

Table 3 displays the results of the proposed approach. For the three samples (*AFRICA*, *OECD*, *GLOBAL*), the criterion BIC_3 estimates six factors corresponding to the estimated value of r in Section 4.2. The results in Table 3 show that countries in the overall sample converged over the period 1975-2008. The p-values associated with the test statistics P_a and P_b are lower than the 1% threshold, indicating the rejection of the null hypothesis of divergence for these countries. Thus, for this sample, the parameter $\hat{\lambda}^*$ is significantly lower than 1 with a value $\hat{\lambda}^* = 0.9421$. The tests based on $t^*(\lambda)$ show that $\lambda \neq 0$ [14] and that the implied value of the parameter β is $\hat{\beta} =$ -0.0266. These results are used to determine the speed of convergence and the half-life τ. The rate of convergence for countries in the sample *GLOBAL* is 5.95% and the corresponding half-life is 26 years.

[TABLE 3 HERE]

The results for OECD countries show that P_a and P_b statistics also accept the hypothesis of convergence for these countries at the 1% level. In addition, there is β-convergence for the OECD countries during the period 1975-2008. With a parameter $\hat{\beta} =$- 0.0306 the speed of convergence is 12.30%, implying a half-life of 22 years, the time necessary for these economies to make up half of the gap separating them from their steady state.

For the AFRICA sample, the null hypothesis of divergence is finally accepted. The probabilities associated with P_a and P_b are higher than the standard thresholds of 5% and 10%.

These results thus point in the same direction as the numerous studies on economic convergence in panel data by accepting the β-convergence for the OECD countries and for the full sample. Moreover, as expected, the treatment of structural change and economic interdependencies led to

[12] The programs are implemented in MATLAB and their elaborations were made possible by using supplementary programs from Pierre Perron, Christophe Hurlin and Serena Ng.

[13] Thus, we obtain 10,000 samples.

[14] Following Evans and Karras (1996) we take for granted that $\lambda \geq 1$.

faster convergence than with the approaches generally used. Estimates of Evans and Karras (1996) over the period 1950-1990 based on a larger sample of 54 rich and poor countries from Summers and Heston's data base provide a rate of convergence of 4.30%. Gaulier et al. (1999) take into account the hypothesis of heterogeneity of the convergence parameter in the procedure of Evans and Karras (1996) and obtain a convergence rate of 11.4% for a sample of 27 OECD countries[15] over the period 1960-1990. However, neither the period considered by these authors nor their sources of data are identical to ours. That can make the comparison more difficult. However, it is important to note that the use of non-stationary panel data, particularly by taking into account the phenomena of co-movement and structural change, substantially solves the problem of bias encountered in cross-sectional analysis, which takes the speed of convergence towards 0. This is the example of the studies by Barro and Sala-i-Martin (1991) and Mankiw et al. (1992) who found a convergence rate of about 2%. As stressed by Evans (1997), in the context of the neoclassical growth model, this slow rate of convergence is incompatible with the fact that physical capital is the only reproducible factor and is paid its marginal product. Because, in the case of slow convergence (for example 2%), the elasticity of output will have to be higher than the observed elasticity for physical capital. In other words, for a more accurate analysis of the convergence process, the use of appropriate procedures such as the one adopted here is necessary.

Concluding remarks

This study has presented a procedure for testing economic convergence in panel data. Based on the approach proposed by Evans and Karras (1996), we applied the procedure based on recent work by Moon and Perron (2004) and by Bai and Ng (2010). The procedure allows us to focus on cross-sectional dependencies and structural changes, which, if ignored, can lead to biases that significantly reduce the power of the test. It appears through the Monte-Carlo experiments that a PANIC based approach controls common factors and structural breaks that may be associated with the convergence process. The study period (1975-2008) is one when sub-regional integration policies were central to economic development strategies in North and South countries alike. However, these policies have caused changes in the structure of the economies by generally permitting them to achieve higher economic growth. Thus, with the persistence of such policies, the poorest economies tend to grow faster.

The approach used to study the convergence process goes beyond the standard approach of considering the phenomena mentioned as simple nuisance parameters. Applications are made on the *AFRICA* sample composed mainly of member countries of the CFA zone and for comparison, on a sample of OECD countries. The results confirm the rejection of the hypothesis of convergence for the countries of sub-Saharan Africa as do studies that have focused on economic convergence in these countries. However, beyond this, an important point emerges. This work has highlighted the fact that the slow rate generally observed in convergence studies is largely due to the omission of certain shocks that affect economies by creating economic co-movements and/or structural changes

[15]The data used by Gaulier et al. (1999) are from the Summers and Heston database.

with significant impacts on the convergence process. This is confirmed by the results for the *OECD* group which validate the assumption of β-convergence for the OECD countries with a relatively high rate of convergence (12.30%). For a heterogeneous sample of 40 rich and poor countries made up of countries in both *AFRICA* and *OECD*, the hypothesis of economic convergence is also accepted at a slower rate than the OECD countries but relatively faster than the convergence measured by existing approaches in the literature.

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APPENDIX

- Construction of test statistics

No interdependence and no break

Im, Pesaran and Shin (1997)

This test also known as IPS test is based on ADF model with a heterogeneous root denoted ρ_i. One of the characteristics is that under the alternative we can have two categories of individuals. Thus, under H_1 we have $\rho_i \prec 0$ for $i = 1, ..., N_1$ and $\rho_i = 0$ for $i = N_1, ..., N$ with $0 \prec N_1 \leq N$. The test statistic W_{t_bar} used in small-sized panels is

$$W_{t\_bar} = \frac{N^{1/2}\left(t\_bar_{NT} - N^{-1}\sum_{i=1}^{N} E\left[t_{iT}|\, \rho_i = 0\right]\right)}{\sqrt{N^{-1}\sum_{i=1}^{N} Var\left[t_{iT}|\, \rho_i = 0\right]}} \to N\left(0, 1\right). \tag{19}$$

where t_bar is the average of individual ADF statistics. $E\left(t_{iT}\right)$ and $Var\left(t_{iT}\right)$ correspond to the mean and variance of the distribution.

Maddala and Wu (1999)

This procedure is based on a combination of p-values of N individual unit root tests carried out independently. It is thus a test of significance *à la* Fisher (1932). The statistic proposed by Maddala and Wu (1999) follows a $\chi^2\left(2N\right)$ and is presented as follows

$$P_{MW} = -2\sum_{i=1}^{N} \ln\left(p_i\right) \tag{20}$$

where p_i corresponds to the p-value associated with the ADF regression for the individual i.

Levin, Lin and Chu (2002)

The test of Levin, Lin and Chu (2002) which is also called test LLC is built on the basis of an adjusted t-statistic (with homogeneous root) and admits a normal distribution

$$t_{\rho}^{*} = \frac{t_{\hat{\rho}}}{\hat{\sigma}_T^{*}} - NT\hat{S}_N\left(\frac{\hat{\sigma}_{\hat{\rho}}}{\hat{\sigma}_{\hat{\xi}}^{2}}\right)\left(\frac{\mu_T^{*}}{\hat{\sigma}_T^{*}}\right) \to N\left(0, 1\right) \tag{21}$$

where μ_T^{*} and $\hat{\sigma}_T^{*}$ correspond respectively to the components for the mean and variance adjustments. Their values are simulated by the authors and depend on T and on the truncation parameter used in the kernel estimation procedure of the long-term variance of the residuals. The term $\hat{S}_N$ is the average of individual ratios of long-run variance of the model to short-run variances of individual residuals for $i = 1, ..., N$.

Interdependence only

Bai and Ng (2004)

For the idiosyncratic component, we used statistics $P^{c}_{\widehat{e}}$, which is standardized in accordance with the procedure of Choi (2001)

$$P^c_{\widehat{e}} = \frac{-2\sum_{i=1}^{N} \log p^c_{\widehat{e}}(i) - 2N}{\sqrt{4N}} \xrightarrow{d} N(0,1) \tag{22}$$

where $p^c_{\hat{e}}(i)$ is the p-value associated with the ADF statistic from the idiosyncratic component of the individual i. With regard to the common component, successive tests of sequences of hypotheses are needed, such as Johansen's (1988) tests for the number of cointegration vectors. The null hypothesis is defined by $H_0 : r_1 = q$ with r_1 the number of common stochastic trends. In this case, the matrix of the q eigenvectors associated with the q first eigenvalues of the covariance matrix of the factors will be denoted $\hat{V}_\perp$ and we can define $\hat{Z}^c_t = \hat{V}'_\perp F^c_t$ where F^c_t is the matrix of the demeaned factors. To determine the first test statistic, we apply the following procedure.

First: We consider the residuals of the first order autoregressive Z^c_t denoted $\hat{\xi}^c_t$ and define[16]

$$\hat{\Sigma}^c_1 = \sum_{j=1}^{J} K(j) \left(T^{-1} \sum_{t=2}^{T} \hat{\xi}^c_{t-j} \hat{\xi}^{c'}_t \right).$$

Second: We can then define $\nu^c_c(q)$, the smallest eigenvalue of

$$\hat{\Phi}^c(q) = 0.5 \left[\sum_{t=2}^{T} (\hat{Z}^c_t \hat{Z}^{c'}_{t-1} + \hat{Z}^c_{t-1} \hat{Z}^{c'}_t) - T(\hat{\Sigma}^c_1 + \hat{\Sigma}^{c'}_1) \right] \left(\sum_{t=2}^{T} \hat{Z}^c_{t-1} \hat{Z}^{c'}_{t-1} \right)^{-1}.$$

Third: From $\nu^c_c(q)$, the statistic is finally defined as

$$MQ^c_c(q) = T\left[\hat{\nu}^c_c(q) - 1\right]. \tag{23}$$

Moon and Perron (2004)

The statistic t^*_b used here is defined in the same way as the statistics P_b (section 2.2.2) and we have

$$t^*_b = \sqrt{N}T\left(\hat{\lambda}^* - 1\right)\sqrt{\frac{1}{NT^2} tr\left(y_{-1} Q_{\hat{\pi}} y'_{-1}\right)} \left(\frac{\hat{\omega}_e}{\hat{\phi}^2_e}\right) \xrightarrow[T,N\to\infty]{} N(0,1). \tag{24}$$

Pesaran (2007)

The null hypothesis of unit root test is $\lambda_i = 1$ for all i against the heterogeneous alternative $\lambda_i \prec 1$ for $i = 1, ..., N_1$ and $\lambda_i = 1$ for $i = N_1 + 1, N_1 + 2, ..., N$. The null hypothesis is tested based on an augmented DF model commonly known as CADF (Cross-sectionally Augmented DF). The statistic $CIPS^*(N,T)$ of Pesaran (2007) used in this paper is the average of the truncated versions of $CADF_i$, denoted $CADF^*_i$

$$CIPS^*(N,T) = \frac{1}{N}\sum_{i=1}^{N} CADF^*_i. \tag{25}$$

Thus, for K_1 and K_2 two positive constants such as $\Pr\left[-K_1 \prec CADF_i \prec K_2\right]$ is sufficiently large (in excess of 0.9999), values of $CADF_i$ smaller than $-K_1$ or larger than K_2 are replaced by their respective bounds. The values of K_1 and K_2 are provided by Pesaran (2007).

[16] $K(j) = 1 - j/(J+1)$ for $j = 0, 1, \ldots, J$.

Break only

Carrion-i-Silvestre et al. (2001)

The null hypothesis of integration of order 1 is tested by generalizing the LM statistic of Harris and Tzavalis (1999). Indeed, this test tends to lead to wrong conclusions in favor of the null hypothesis of non-stationarity due to the bias generated by misspecification of the deterministic component through the omission of a structural change affecting the mean. In the case of such omission, the estimator of λ obtained from the null model has a bias which we denote by B_N, which is a function of T and the break fraction $\alpha = T_b/T$. Thus, under the null hypothesis, $\hat{\lambda}_0 - 1$ is a nonzero constant and we have

$$\underset{N\to\infty}{p\lim} \frac{1}{N}\left(\hat{\lambda}_0 - 1\right) = B_N. \quad (26)$$

The test statistic is thus different from that of Harris and Tzavalis (1999) by the fact that the parameter λ used is adjusted using the bias correction

$$B_N = \frac{-3T\,(T-3)}{(1+2\alpha^2-2\alpha)\,T + (2\alpha-2)\,T - 1}.$$

Interdependence and break

Bai and Carrion-i-Silvestre (2009)

The test P_m of Bai and Carrion-i-Silvestre (2009) is based on the model (9). P_m is a Choi (2001) type statistic and we have

$$P_m = \frac{-2\sum_{i=1}^{N} \ln p_i - 2N}{\sqrt{4N}} \to N(0,1); \quad (27)$$

where p_i is the p-value associated with the statistic

$$\text{MSB}_i = \frac{T^{-2}\sum_{t=1}^{T} \hat{e}_{i,t-1}^2}{\hat{\sigma}_{\varepsilon,i}} \quad (28)$$

corresponding to the individual i. $\hat{\sigma}_{\varepsilon,i}$ is the long-term variance of ε_{it} defined by the relation $e_{it} = \lambda e_{i,t-1} + \varepsilon_{it}$.

- Tables

Table 1: Simulation results

	(N,T)		Experiment 1			Experiment 2		
			P_a	P_b	mean r	P_a	P_b	mean r
Size								
	(20,50)	True r	10.5	6.8	1.00	10.7	7.0	1.00
		BIC_3	10.5	6.8	1.00	10.7	7.0	1.00
		IC_1	10.5	6.8	1.00	10.7	7.0	1.00
	(20,100)	True r	10.1	6.8	1.00	10.6	6.9	1.00
		BIC_3	10.1	6.8	1.00	10.6	6.9	1.00
		IC_1	10.1	6.8	1.00	10.6	6.9	1.00
	(50,50)	True r	9.4	6.7	1.00	9.3	6.6	1.00
		BIC_3	9.4	6.7	1.00	9.3	6.6	1.00
		IC_1	9.4	6.7	1.00	9.3	6.6	1.00
	(50,100)	True r	8.3	6.0	1.00	8.2	5.9	1.00
		BIC_3	8.3	6.0	1.00	8.2	5.9	1.00
		IC_1	8.3	6.0	1.00	8.2	5.9	1.00
Power								
	(20,50)	True r	99.8	99.5		99.8	99.3	
		BIC_3	99.8	99.5		99.8	99.3	
		IC_1	99.8	99.5		99.8	99.3	
	(20,100)	True r	100	100		100	100	
		BIC_3	100	100		100	100	
		IC_1	100	100		100	100	
	(50,50)	True r	100	100		100	100	
		BIC_3	100	100		100	100	
		IC_1	100	100		100	100	
	(50,100)	True r	100	100		100	100	
		BIC_3	100	100		100	100	
		IC_1	100	100		100	100	

Notes: For Size, P_a and P_b columns give the percentage of replications in which the null hypothesis of a unit root is rejected at the 5% level. The number of factors is either set to 1 (the true number) or estimated using the information criteria suggested by Bai and Ng (2002). The last two columns provide the mean number of estimated factors. For Power, entries represent the percentage of replications in which the null hypothesis of a unit root is rejected.

Table 2: Results based on different generations of tests

Assumptions taken into account :	AFRICA	OECD	GLOBAL
No interdependence, no break			
Im, Pesaran and Shin (1997)			
W_{tbar}	-1.1899	-1.5065*	-1.7529**
Maddala and Wu (1999)			
P_{MW}	41.3962	56.8904**	94.0668
Levin, Lin and Chu (2002)			
t^*_ρ	-1.6285*	-4.5233***	-7.1853***
Interdependence only			
Bai and Ng (2004)			
$(P^c_{\hat{e},Choi}\ ;MQ_c)$	(0.32; -19.197)	(3.39*** ; -21.95)	(2.24** ; -19.55)
Moon and Perron (2004)			
t^*_b	-5.61***	-5.35***	-8.57***
Pesaran (2007)			
$CIPS^*$	-1.614	-1.530	-1.269
Break only			
Carrion, Barrio and López (2001)			
$stat_{CDL}$	-1.121	2.662	0.155
Interdependence and break			
Bai and Carrion-i-Silvestre (2009)			
P_m	-4.022***	-4.312***	-6.189***

Notes: The signs (*), (**) and (***) indicate significance levels respectively equal to 10% 5% and 1% . $stat_{CDL}$ is the statistic of Carrion-i-Silvestre et al. (2001). Except for the test of Pesaran (2007) where the number of common factors is set to 1, the second generation tests (Bai and Ng 2004, Moon and Perron 2004) include a number of common factors (selected by the criterion BIC_3) which is equal to 6 for the three samples. This is also true for the test of Bai and Carrion-i-Silvestre (2009). According to the maximum number of factors allowed, it is equal to 8.

Table 3: Estimation results of the proposed approach

Samples :	*AFRICA*	*OECD*	*GLOBAL*
Stochast. converg. ($H_0 : \lambda = 1$)			
P_a	0.4768	-7.5155***	-5.8031***
	(0.6832)	(0.0000)	(0.0000)
P_b	0.4555	-3.8373***	-3.9214***
	(0.6756)	(0.0000)	(0.0000)
Analysing β-convergence			
$\hat{\lambda}^*$	1.00	0.8843***	0.9421***
$t^*(\lambda)$		55.95	120.02
Crit. val. (5%)		1.95	1.94
$\hat{\beta}$		-0.0306	-0.0266
$\hat{\theta}$		12.30%	5.95%
$\hat{\tau}$		22 years	26 years

Notes : The parameter $\hat{\beta}$ is equal to $\hat{\beta} = ((\hat{\lambda}^*)^T - 1)/T$. $\hat{\theta}$ is the convergence rate given by $\hat{\theta} = -\ln(1 + \hat{\beta}T)/T$. $\hat{\tau}$ corresponds to the half-life (in years) given by $\hat{\tau} = -\ln(2)/\ln(1 + \hat{\beta})$. Crit. val. (5%) is the critical value of $t^*(\lambda)$ test at 5% level. Values in parentheses represent the p-values.

Table 4: Structural change tests

			cross-section demeaned log GDP per capita					
	Log GDP per capita		AFRICA		OECD		GLOBAL	
Countries	Break	Date	Break	Date	Break	Date	Break	Date
Australia					+	1982		
Austria					+	2000	+	1989
Belgium					+	1997		
Canada					+	1989	+	1992
Danemark					+	2000		
Finland								
France								
Germany					+	1999		
Greece								
Ireland					+	1995		
Italy								
Japan								
Netherlands					+	1990		
New Zealand					+	1987	+	1992
Norway								
Portugal					+	1987		
South Africa								
Spain							+	1989
Sweden					+	1990		
United Kingdom								
United States					+	2000		
Benin	+	1997	+	1991				
Burkina Faso							+	1995
Cameroon								
Central African Rep.	+	1989	+	2002				
Chad	+	2003	+	2002				
Congo, Dém.			+	1992				
Congo, Rep.	+	1980	+	1980				
Côte d'Ivoire								
Gabon								
Gambia			+	1989			+	1994
Ghana								
Guinea Bissau	+	2001					+	2001
Liberia	+	1989	+	1989			+	1989
Mali	+	2000	+	1991			+	1981
Niger	+	1983	+	1983				
Nigeria							+	1980
Senegal	+	2003	+	1991			+	1982
Sierra Leone								
Togo	+	1982						

Notes : The break dates are estimated following the procedure of Bai and Perron (1998). We consider the case of a single structural change. The sign (+) indicates the presence of a break.

Table 5: Cross-section Dependence (CD) test

	CD statistics					
	Log GDP per capita			cross-section demeaned log GDP per capita		
Regressions $ADF(p)$	$p=1$	$p=2$	$p=3$	$p=1$	$p=2$	$p=3$
AFRICA	3.70	3.40	3.67	-0.30	0.24	-0.36
OECD	24.63	24.32	24.26	-3.00	-2.63	-2.65
GLOBAL	17.04	15.96	15.01	8.17	7.56	6.88

Notes: CD is the statistic of Pesaran (2004). The test statistic is the average of pair-wise Pearson's correlation coefficients of the residuals obtained from ADF-type regression equations. We consider different orders p. The statistic is compared to the standard normal distribution. The null hypothesis of independence is rejected if $|CD| \geq 1.96$.

Table 6: Individual CADF tests

	Log GDP per capita	cross-section demeaned log GDP per capita		
Pays		AFRICA	OECD	GLOBAL
Australia	1.496		-1.612	-0.370
Austria	-0.336		-3.251*	-1.180
Belgium	-0.138		-0.651	-0.901
Canada	0.358		-1.100	-1.548
Danemark	-0.460		-0.547	-1.012
Finland	-1.874		-1.861	-1.679
France	0.262		-0.811	-0.701
Germany	-0.896		-2.505	-1.050
Greece	-1.257		-2.293	1.453
Italy	-0.802		-0.848	-0.824
Ireland	0.112		-1.079	-1.199
Japan	-1.609		-2.146	-1.370
Netherlands	-0.258		-3.153*	0.370
New Zealand	0.092		-0.722	-1.267
Norway	-0.153		-1.208	-1.383
Portugal	-0.690		-1.654	-0.726
South Africa	-0.425	-1.165		-1.227
Spain	-0.321		-1.202	-1.439
Sweden	-0.412		-1.162	-1.033
United Kingdom	0.575		-1.475	0.181
United States	0.087		-1.324	-0.653
Benin	-2.670	-3.070*		-2.487
Burkina Faso	1.303	-2.062		-0.972
Cameroun	-2.229	-2.687		-0.773
Central African Rep.	-1.134	-0.027		-1.464
Chad	-5.749**	-0.914		-3.692**
Congo, Dem.	0.579	-0.849		-1.039
Congo, Rep.	-3.292*	-3.811**		-3.262*
Côte d'Ivoire	-3.384**	-1.184		-2.323
Gabon	-1.122	-0.972		-0.946
Gambia	0.235	-1.129		-1.628
Ghana	-3.179*	-0.041		-2.472
Guinea Bissau	-1.522	-2.268		0.056
Liberia	-1.218	-1.739		0.279
Mali	-2.564	0.180		-2.493
Niger	-1.212	-1.495		-1.659
Nigeria	-3.436**	-2.143		-3.653**
Senegal	-2.038	-1.716		-1.853
Sierra Leone	-0.844	-2.402		-1.728
Togo	-1.323	-2.787		-1.073

Notes : The critical values at 5% and 10% are respectively equal to -3.34 and -2.96 for the samples *OECD* and *AFRICA*. For the sample *GLOBAL*, the critical values to the respective thresholds of 5% and 10% are -3.34 and -2.97. The signs (*) and (**) indicate respectively 10% and 5% significance levels.